Strong one-pion decay of ground state charmed baryons

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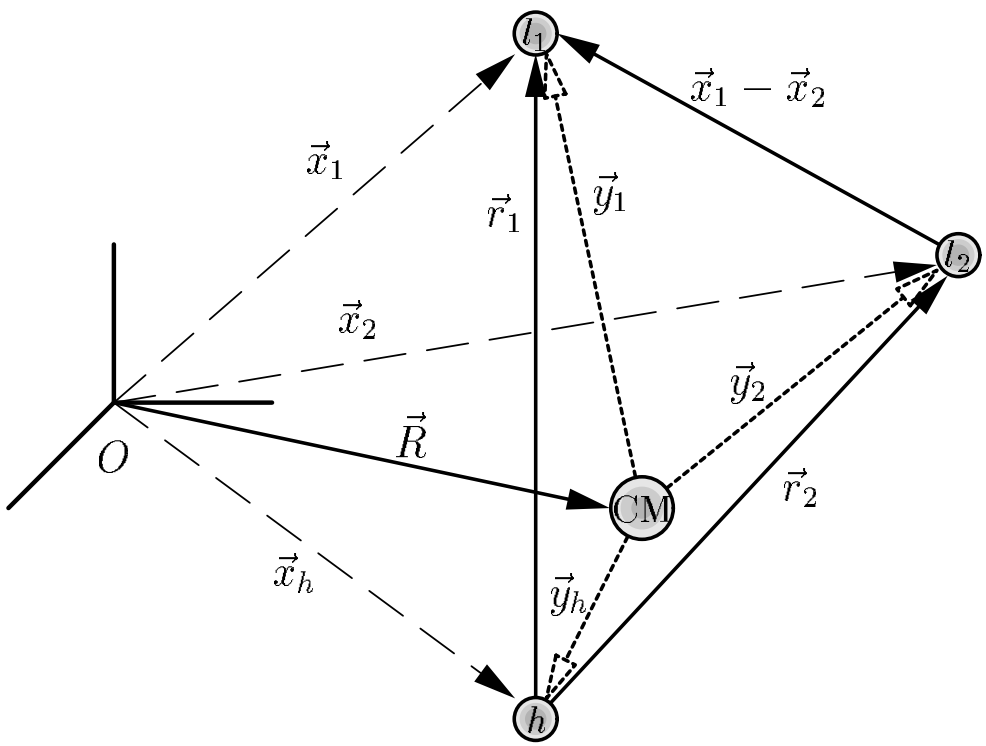
Outline of the talk

- Baryon description
  - Three–body Hamiltonian
  - Quark–quark potentials
  - Orbital wave functions
  - Baryon states
- One–pion emission amplitude model
- Results
  - $\Sigma_c \to \Lambda_c \, \pi$
  - $\Sigma_c^* \to \Lambda_c \, \pi$
  - $\Xi_c^* \to \Xi_c \, \pi$
- Summary

Three–body Hamiltonian

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}} + H_{int} \qquad H_{int} = \overline{M} + \sum_{j=1,2} H_j^{sp} + V_{l_1,l_2}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_h}$$

$$H_j^{sp} = -\frac{\vec{\nabla}_j^2}{2\mu_j} + V_{h,l_j}(\vec{r}_j, spin), \qquad \overline{M} = m_{l_1} + m_{l_2} + m_h, \qquad \frac{1}{\mu_j} = \frac{1}{m_{l_j}} + \frac{1}{m_h}$$



Quark–quark Potentials

We use 5 different quark–quark potentials taken from

B. Silvestre-Brac, Few-Body Systems 20, 1 (1996)

R.K. Bhaduri, L.E. Cohler, Y. Nogami, Nuovo Cimento A65, 376 (1981)

Common structure:

- Confinement term
- OGE
  - Coulomb term
  - Hyperfine terms

$$V_{ij}^{q\bar{q}}(r) = -\frac{\kappa\left(1-e^{-r/r_c}\right)}{r} + \lambda r^p - \Lambda$$

$$+\left\{a_0 \frac{\kappa}{m_i m_j} \frac{e^{-r/r_0}}{r r_0^2} + \frac{2\pi}{3 m_i m_j} \kappa' \left(1 - e^{-r/r_c}\right) \frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3}\right\} \vec{\sigma}_i \vec{\sigma}_j$$

$$V_{ij}^{qq}(r) = \frac{1}{2} V_{ij}^{q\bar{q}}(r)$$

Differences:

- $p = 1$ (lattice QCD) or $p = 2/3$ (Regge trajectories)
- Form factors in OGE terms

Parameters adjusted to light and heavy-light meson spectroscopy

Orbital wave functions

Variational Ansazt $\phi^{S_l}_{l_1,l_2,h}(\vec{r}_1,\vec{r}_2) = N\phi^h_{l_1}(r_1)\phi^h_{l_2}(r_2)F^{S_l}_{l_1,l_2}(r_{12})$

- S_l: Total spin of the light degrees of freedom assumed to be fixed (Heavy Quark Symmetry limit)
- $\phi^h_{l_j}(r_j) = (1+\alpha_j\, r_j)\,\psi^h_{l_j}(r_j)$
  Ground state wave function $\psi^h_{l_j}(r_j)$ for the relative motion of the light–quark heavy–quark system corrected at large distances
- Jastrow correlation function

$$F^{S_l}_{l_1,l_2}(r_{12}) = f^{S_l}_{l_1,l_2}(r_{12})\sum_{j=1}^{4} a_j\, e^{-b_j^2\,(r_{12}+d_j)^2} \qquad a_1 = 1$$

$$f^{S_l}_{l_1,l_2}(r_{12}) = \begin{cases} 1-e^{-c\,r_{12}} & \text{if} \quad V_{l_1,l_2}(r_{12}=0,S_l) \gg 0 \\ 1 & \text{if} \quad V_{l_1,l_2}(r_{12}=0,S_l) \leq 0 \end{cases}$$

In momentum space:

$$\widetilde{\phi}^{S_l}_{l_1,l_2,h}(\vec{Q}_1,\vec{Q}_2) \quad = \quad \frac{1}{(2\pi)^{3/2}}\int d^3P\; N\,\widetilde{\phi}^h_{l_1}(|\vec{P}|)\;\widetilde{\phi}^h_{l_2}(|\vec{Q}_1+\vec{Q}_2-\vec{P}|)\;\widetilde{F}^{S_l}_{l_1,l_2}(|\vec{Q}_1-\vec{P}|)$$

Baryon states

$$\left|B, s\,\vec{P}\,\right\rangle_{NR} = \int d^3Q_1 \int d^3Q_2\, \frac{1}{\sqrt{2}} \sum_{\alpha_1,\alpha_2,\alpha_3} \frac{\hat{\psi}^{(B,s)}_{\alpha_1,\alpha_2,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,)}{(2\pi)^3\,\sqrt{2E_{f_1}(|\vec{p}_1|)2E_{f_2}(|\vec{p}_2|)2E_{f_3}(|\vec{p}_3|)}}$$

$$\times \left|\,\alpha_1\,\vec{p}_1 = \frac{m_{f_1}}{\overline{M}}\vec{P} + \vec{Q}_1\,\right\rangle \left|\,\alpha_2\,\vec{p}_2 = \frac{m_{f_2}}{\overline{M}}\vec{P} + \vec{Q}_2\,\right\rangle \left|\,\alpha_3\,\vec{p}_3 = \frac{m_{f_3}}{\overline{M}}\vec{P} - \vec{Q}_1 - \vec{Q}_2\,\right\rangle$$

with

$$\alpha_j \equiv (s_j, f_j, c_j), \qquad \langle \alpha'\vec{p}\,'|\alpha\,\vec{p}\rangle = \delta_{\alpha,\,\alpha'}\,(2\pi)^3\,2E(|\vec{p}|)\,\delta^3(\vec{p}-\vec{p}\,')$$

The normalization is given by

$${}_{NR}\langle B, s'\vec{P}\,'|B, s\vec{P}\,\rangle_{NR} = \delta_{s,\,s'}\,(2\pi)^3\,\delta^3(\vec{P}-\vec{P}\,')$$

An example:

$$\hat{\psi}^{(\Xi^0_c,s)}_{\alpha_1,\,\alpha_2,\,\alpha_3}(\,\vec{Q}_1,\vec{Q}_2\,) = \frac{\varepsilon_{c_1\,c_2\,c_3}}{\sqrt{3!}}\,(1/2,1/2,0;s_1,s_2,0)$$

$$\times\,\frac{\delta_{s_3,\,s}\,\delta_{f_3,\,c}}{\sqrt{2}}\,\left(\delta_{f_1,\,d}\,\delta_{f_2,\,s}\,\widetilde{\phi}^{S_l=0}_{d,s,c}(\,\vec{Q}_1,\vec{Q}_2\,) - \delta_{f_1,\,s}\,\delta_{f_2,\,d}\,\widetilde{\phi}^{S_l=0}_{s,d,c}(\,\vec{Q}_1,\vec{Q}_2\,)\right)$$

Pion emission amplitude

PCAC:

$$\partial^\mu \, J^{d\,u}_{A\,\mu}(x) = \, f_\pi \, m_\pi^2 \, \Phi_{\pi^-}(x) \quad , \quad f_\pi = 130.7\text{MeV}$$

- Matrix elements

$$\left\langle B', \; s' \, \vec{P}_{B'} \; \middle| \, q^\mu \, J^{d\,u}_{A\,\mu}(0) \, \middle| \; B, \; s \, \vec{P}_B \right\rangle = -i \, f_\pi \, \frac{m_\pi^2}{q^2 - m_\pi^2} \, \mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$$

$\mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$: Pion emission amplitude

- Pion–pole contribution

$$\left\langle B', \; s' \, \vec{P}_{B'} \, | \, q^\mu \, J^{d\,u}_{A\,\mu}(0) \, | \, B, \; s \, \vec{P}_B \right\rangle_{pion-pole} = -i \, f_\pi \, \frac{q^2}{q^2 - m_\pi^2} \, \mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$$

- Non–pole contribution

$$\left\langle B', \; s' \, \vec{P}_{B'} \, | \, q^\mu \, J^{d\,u}_{A\,\mu}(0) \, | \, B, \; s \, \vec{P}_B \right\rangle_{non-pole} = i \, f_\pi \, \mathcal{A}^{(s,s')}_{BB'\pi}(P_B, P_{B'})$$

$\Sigma_c \to \Lambda_c \pi$ decay

$$\mathcal{A}^{(s,s')}_{\Sigma_c^{++}\Lambda_c^+\pi^+}(P,P') = \frac{-i}{f_\pi} \left\langle \Lambda_c^+,\ s'\,\vec{P}' \,\middle|\, q^\mu\, J^{d\,u}_{A\,\mu}(0) \,\middle|\, \Sigma_c^{++},\ s\,\vec{P} \right\rangle_{non-pole}$$

$$= i g_{\Sigma_c^{++}\Lambda_c^+\pi^+}\ \overline{u}_{\Lambda_c^+\ s'}(\vec{P}')\ \gamma_5\ u_{\Sigma_c^{++}\ s}(\vec{P})$$

$$\Gamma(\Sigma_c^{++} \to \Lambda_c^+\pi^+) = \frac{|\vec{q}|}{8\pi M^2_{\Sigma_c^{++}}}\ g^2_{\Sigma_c^{++}\Lambda_c^+\pi^+} \left((M_{\Sigma_c^{++}} - M_{\Lambda_c^+})^2 - m_\pi^2 \right)$$

Using $\vec{P}' = -|\vec{q}|\hat{k}$ and taking into account the different normalization of our NR states

$$g_{\Sigma_c^{++}\Lambda_c^+\pi^+} = \frac{-1}{f_\pi}\ \frac{\sqrt{E_{\Lambda_c^+}(|\vec{q}|) + M_{\Lambda_c^+}}\sqrt{2M_{\Sigma_c^{++}} 2E_{\Lambda_c^+}(|\vec{q}|)}}{|\vec{q}|\sqrt{2M_{\Sigma_c^{++}}}}$$

$$\times \left((M_{\Sigma_c^{++}} - E_{\Lambda_c^+}(|\vec{q}|))\ A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ 0} + |\vec{q}|\ A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ 3} \right)$$

$$A^{1/2,1/2}_{\Sigma_c^{++}\Lambda_c^+,\ \mu} = {}_{NR}\left\langle \Lambda_c^+,\ 1/2\ -|\vec{q}|\,\vec{k} \ \middle|\ J^{d\,u}_{A\,\mu}(0) \ \middle|\ \Sigma_c^{++},\ 1/2\ \vec{0} \right\rangle_{NR,\ non-pole}$$

$\Sigma_c \to \Lambda_c \pi$ decay: Results

	$g_{\Sigma_c^{++}\Lambda_c^+\pi^+}$	$\Gamma(\Sigma_c^{++} \to \Lambda_c^+\pi^+)$ [MeV]	$\Gamma(\Sigma_c^{+} \to \Lambda_c^+\pi^0)$ [MeV]	$\Gamma(\Sigma_c^{0} \to \Lambda_c^+\pi^-)$ [MeV]
This work	$21.73 \pm 0.32 \pm 0.08$	$2.41 \pm 0.07 \pm 0.02$	$2.79 \pm 0.08 \pm 0.02$	$2.37 \pm 0.07 \pm 0.02$
Experiment		$2.3 \pm 0.2 \pm 0.3$ [1)]	< 4.6 (CL=90%) [2)]	$2.5 \pm 0.2 \pm 0.3$ [1)]
		$2.05^{+0.41}_{-0.38} \pm 0.38$ [3)]		$1.55^{+0.41}_{-0.37} \pm 0.38$ [3)]
Theory				
CQM		1.31 ± 0.04 [4)]	1.31 ± 0.04 [4)]	1.31 ± 0.04 [4)]
		$2.025^{+1.134}_{-0.987}$ [5)]		$1.939^{+1.114}_{-0.954}$ [5)]
HHCPT	$22, 29.3$ [6)]	$2.47, 4.38$ [6)]	$2.85, 5.06$ [6)]	$2.45, 4.35$ [6)]
		2.5 [7)]	3.2 [7)]	2.4 [7)]
				1.94 ± 0.57 [8)]
LFQM		1.64 [9)]	1.70 [9)]	1.57 [9)]
RTQM		2.85 ± 0.19 [10)]	3.63 ± 0.27 [10)]	2.65 ± 0.19 [10)]

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4) J. L. Rosner, Phys. Rev. D52, 6461 (1995)
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6) T. M. Yan *et al.*, Phys. Rev. D46, 1148 (1992)
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9) S. Tawfig, P. J. O'Donnell, J. G. Körner, Phys. Rev. D56, 054010 (1998)
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$\Sigma_c^* \to \Lambda_c \pi$ decay

$$\mathcal{A}^{(s,s')}_{\Sigma_c^{*++}\Lambda_c^+\pi^+}(P,P') = \frac{-i}{f_\pi}\left\langle \Lambda_c^+,\ s'\,\vec{P}' \,\middle|\, q^\mu\, J^{d\,u}_{A\,\mu}(0) \,\middle|\, \Sigma_c^{*++},\ s\,\vec{P}\right\rangle_{non-pole}$$

$$= i\frac{g_{\Sigma_c^{*++}\Lambda_c^+\pi^+}}{2M_{\Lambda_c^+}}\, q_\nu\, \overline{u}_{\Lambda_c^+\ s'}(\vec{P}')\, u^\nu_{\Sigma_c^{*++}\ s}(\vec{P})$$

$$\Gamma(\Sigma_c^{*++} \to \Lambda_c^+\pi^+) = \frac{|\vec{q}|^3}{24\pi M^2_{\Sigma_c^{*++}}}\,\frac{g^2_{\Sigma_c^{*++}\Lambda_c^+\pi^+}}{4M^2_{\Lambda_c^+}}\left((M_{\Sigma_c^{*++}} + M_{\Lambda_c^+})^2 - m_\pi^2\right)$$

$$g_{\Sigma_c^{*++}\Lambda_c^+\pi^+} = \frac{\sqrt{3}}{f_\pi\sqrt{2}}\,\frac{2M_{\Lambda_c^+}\sqrt{2M_{\Sigma_c^{*++}}2E_{\Lambda_c^+}(|\vec{q}|)}}{|\vec{q}|\sqrt{2M_{\Sigma_c^{*++}}\left(E_{\Lambda_c^+}(|\vec{q}|) + M_{\Lambda_c^+}\right)}}$$

$$\times\left((M_{\Sigma_c^{*++}} - E_{\Lambda_c^+}(|\vec{q}|))\, A^{1/2,1/2}_{\Sigma_c^{*++}\Lambda_c^+,\,0} + |\vec{q}|\, A^{1/2,1/2}_{\Sigma_c^{*++}\Lambda_c^+,\,3}\right)$$

$$A^{1/2,1/2}_{\Sigma_c^{*++}\Lambda_c^+,\,\mu} = {}_{NR}\left\langle \Lambda_c^+,\ 1/2\ -|\vec{q}|\,\vec{k} \,\middle|\, J^{d\,u}_{A\,\mu}(0) \,\middle|\, \Sigma_c^{*++},\ 1/2\,\vec{0}\right\rangle_{NR,\ non-pole}$$

$\Sigma_c^* \to \Lambda_c \pi$ decay: Results

	$g_{\Sigma_c^{*++}\Lambda_c^+\pi^+}$	$\Gamma(\Sigma_c^{*++} \to \Lambda_c^+\pi^+)$ [MeV]	$\Gamma(\Sigma_c^{*+} \to \Lambda_c^+\pi^0)$ [MeV]	$\Gamma(\Sigma_c^{*0} \to \Lambda_c^+\pi^-)$ [MeV]
This work	$36.20 \pm 0.75 \pm 0.13$	$17.52 \pm 0.74 \pm 0.12$	$17.31 \pm 0.73 \pm 0.12$	$16.90 \pm 0.71 \pm 0.12$
Experiment		$14.1^{+1.6}_{-1.5} \pm 1.4$ [11]	< 17 (CL=90%) [2]	$16.6^{+1.9}_{-1.7} \pm 1.4$ [11]
Theory				
QCDSR	$13.8 \div 24.2$ [12]			
	$32.5 \pm 2.1 \pm 6.9$ [13]			
CQM		20 [4]	20 [4]	20 [4]
HHCPT		25 [7]	25 [7]	25 [7]
LFQM		12.84 [9]		12.40 [9]
RTQM		21.99 ± 0.87 [10]		21.21 ± 0.81 [10]

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$\Xi_c^* \longrightarrow \Xi_c \pi$ decay

$$\mathcal{A}^{(s,s')}_{\Xi_c^{*\,+}\Xi_c^0\pi^+}(P,P') = \frac{-i}{f_\pi}\left\langle \Xi_c^0,\ s'\,\vec{P}'\,|\,q^\mu\,J^{d\,u}_{A\,\mu}(0)\,|\,\Xi_c^{*\,+},\ s\,\vec{P}\right\rangle_{non-pole}$$

$$= i\,\frac{g_{\Xi_c^{*\,+}\Xi_c^+\pi^+}}{M_{\Xi_c^+}+M_{\Xi_c^0}}\,q_\nu\,\overline{u}_{\Xi_c^0\ s'}(\vec{P}')\,u^\nu_{\Xi_c^{*\,+}\ s}(\vec{P})$$

$$\Gamma(\Xi_c^{*\,+}\to\Xi_c^0\pi^+) = \frac{|\vec{q}|^3}{24\pi M^2_{\Xi_c^{*\,+}}}\,\frac{g^2_{\Xi_c^{*\,+}\Xi_c^0\pi^+}}{(M_{\Xi_c^+}+M_{\Xi_c^0})^2}\left((M_{\Xi_c^{*\,+}}+M_{\Xi_c^0})^2-m_\pi^2\right)$$

$$g_{\Xi_c^{*\,+}\Xi_c^0\pi^+} = \frac{\sqrt{3}}{f_\pi\sqrt{2}}\,\frac{(M_{\Xi_c^+}+M_{\Xi_c^0})\sqrt{2M_{\Xi_c^{*\,+}}2E_{\Xi_c^0}(|\vec{q}|)}}{|\vec{q}|\sqrt{2M_{\Xi_c^{*\,+}}\left(E_{\Xi_c^0}(|\vec{q}|)+M_{\Xi_c^0}\right)}}$$

$$\times\left((M_{\Xi_c^{*\,+}}-E_{\Xi_c^0}(|\vec{q}|))\,A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\ 0}+|\vec{q}|\,A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\ 3}\right)$$

$$A^{1/2,1/2}_{\Xi_c^{*\,+}\Xi_c^0,\ \mu} = {}_{NR}\left\langle \Xi_c^0,\ 1/2\ -|\vec{q}|\,\vec{k}\,|\ J^{d\,u}_{A\,\mu}(0)\,|\,\Xi_c^{*\,+},\ 1/2\ \vec{0}\right\rangle_{NR,\ non-pole}$$

$\Xi_c^* \longrightarrow \Xi_c \pi$ decay: Results

	$g_{\Xi_c^{*+}\Xi_c^0\pi^+}$	$\Gamma(\Xi_c^{*+} \to \Xi_c^0\pi^+)$ [MeV]	$\Gamma(\Xi_c^{*+} \to \Xi_c^+\pi^0)$ [MeV]	$\Gamma(\Xi_c^{*0} \to \Xi_c^+\pi^-)$ [MeV]	$\Gamma(\Xi_c^{*0} \to \Xi_c^0\pi^0)$ [MeV]
This work	$-28.83 \pm 0.50 \pm 0.10$	$1.84 \pm 0.06 \pm 0.01$	$1.34 \pm 0.04 \pm 0.01$	$2.07 \pm 0.07 \pm 0.01$	$0.956 \pm 0.030 \pm 0.007$
Theory					
LFQM		1.12[9]	0.69[9]	1.16[9]	0.72[9]
RTQM		1.78 ± 0.33[10]	1.26 ± 0.17[10]	2.11 ± 0.29[10]	1.01 ± 0.15[10]

	$\Gamma(\Xi_c^{*+} \to \Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ [MeV]	$\Gamma(\Xi_c^{*+} \to \Xi_c^0\pi^+ + \Xi_c^+\pi^0)$ [MeV]
This work	$3.18 \pm 0.10 \pm 0.01$	$3.03 \pm 0.10 \pm 0.01$
Exp.	< 3.1 (CL=90%)[14]	< 5.5 (CL=90%)[15]
Theory		
CQM	$< 2.3 \pm 0.1$[4]	$< 2.3 \pm 0.1$[4]
	$1.191 \div 3.971$[5]	$1.230 \div 4.074$[5]
HHCPT	2.44 ± 0.85[8]	2.51 ± 0.88[8]
LFQM	1.81[9]	1.88[9]
RTQM	3.04 ± 0.50[10]	3.12 ± 0.33[10]

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Summary

- First dynamical calculation of these observables within a non relativistic quark model
- Wave functions
  - Obtained using a variational ansazt made possible by HQS constraints
- One–pion emision amplitude
  - We use PCAC to relate the one–pion emission amplitude to weak current matrix elements
- Results
  - Stable against different quark-quark interaction
  - Overall good agreement with experiment