A possible nuclear effect on the NuTeV $\sin^2 \theta_W$ anomaly

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References

Modifications of parton distribution functions (PDFs) in nuclei

(1) M. Hirai, SK, M. Miyama, Phys. Rev. D64 (2001) 034003.

(2) M. Hirai, SK, T.-H. Nagai, Phys. Rev. C70 (2004) 044905.

NPDF codes could be obtained from
http://research.kek.jp/people/kumanos/nuclp.html

Nuclear PDF effects on $\sin^2\theta_W$

(1) SK, Phys. Rev. D66 (2002) 111301.

(2) M. Hirai, SK, T.-H. Nagai, Phys. Rev. D71 (2005) 113007.

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Introduction: Paschos-Wolfenstein (PW) relation and $\sin^2\theta_W$

Charged current (CC) cross sections for νN and $\bar{\nu} N$: N = isoscalar nucleon

$$\frac{d\sigma_{CC}^{\nu N}}{dx\,dy} = \sigma_0 x\left[d(x) + s(x) + \{\bar{u}(x) + \bar{c}(x)\}(1-y)^2\right]$$

$$\frac{d\sigma_{CC}^{\bar{\nu} N}}{dx\,dy} = \sigma_0 x\left[\bar{d}(x) + \bar{s}(x) + \{u(x) + c(x)\}(1-y)^2\right]$$

where $\sigma_0 = \dfrac{G_F^{\,2} s}{\pi}$

$$u_L = +\frac{1}{2} - \frac{2}{3}\sin^2\theta_W, \quad u_R = -\frac{2}{3}\sin^2\theta_W$$

$$d_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \quad d_R = +\frac{1}{3}\sin^2\theta_W$$

Neutral current (NC) :

$$\frac{d\sigma_{NC}^{\nu N}}{dx\,dy} = \sigma_0 x\Big[\{u_L^{\,2} + u_R^{\,2}(1-y)^2\}\{u(x) + c(x)\} + \{u_R^{\,2} + u_L^{\,2}(1-y)^2\}\{\bar{u}(x) + \bar{c}(x)\}$$
$$+\{d_L^{\,2} + d_R^{\,2}(1-y)^2\}\{d(x) + s(x)\} + \{d_R^{\,2} + d_L^{\,2}(1-y)^2\}\{\bar{d}(x) + \bar{s}(x)\}\Big]$$

$$\frac{d\sigma_{NC}^{\bar{\nu} N}}{dx\,dy} = \frac{d\sigma_{NC}^{\nu N}}{dx\,dy}(L \leftrightarrow R)$$

PW relation : $R^- = \dfrac{\sigma_{NC}^{\nu N} - \sigma_{NC}^{\bar{\nu} N}}{\sigma_{CC}^{\nu N} - \sigma_{CC}^{\bar{\nu} N}} = \dfrac{1}{2} - \sin^2\theta_W$

"NuTeV anomaly"

NuTeV: $\sin^2\theta_W$ = 0.2277 ± 0.0013 (stat) ± 0.0009 (syst)

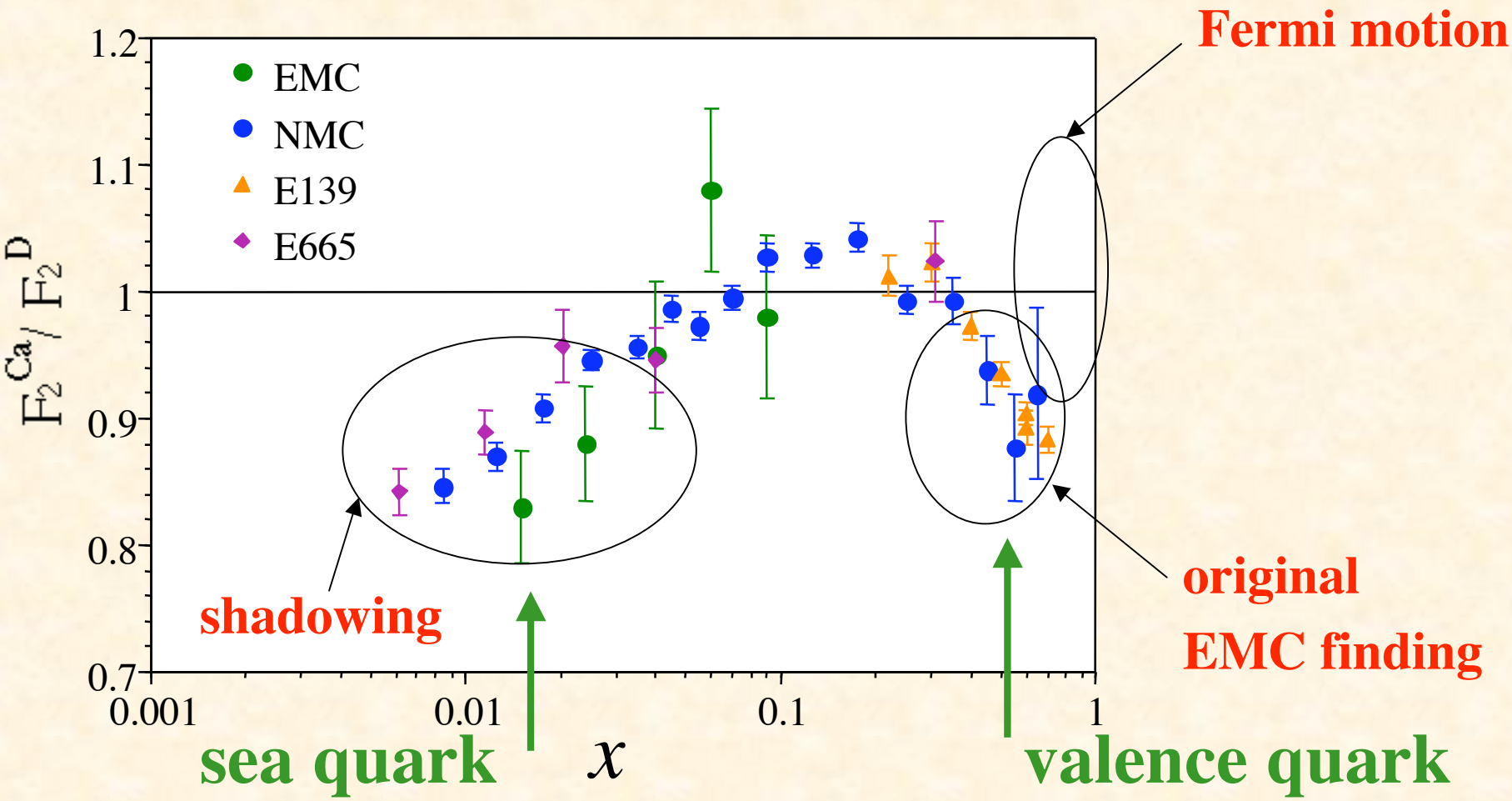
Others: $\sin^2\theta_W = 1 - m_W^2/m_Z^2$ = 0.2227 ± 0.0004

Actual target is iron nucleus in the NuTeV experiment!

Nuclear modification

$$F_2^A = \sum_i e_i^2 \, x \left[q_i(x) + \bar{q}_i(x) \right]_A$$

Nuclear modification of F_2^A / F_2^D is well known in electron/muon scattering.



Experimental data: total number=951

(1) F_2^A / F_2^D 606 data

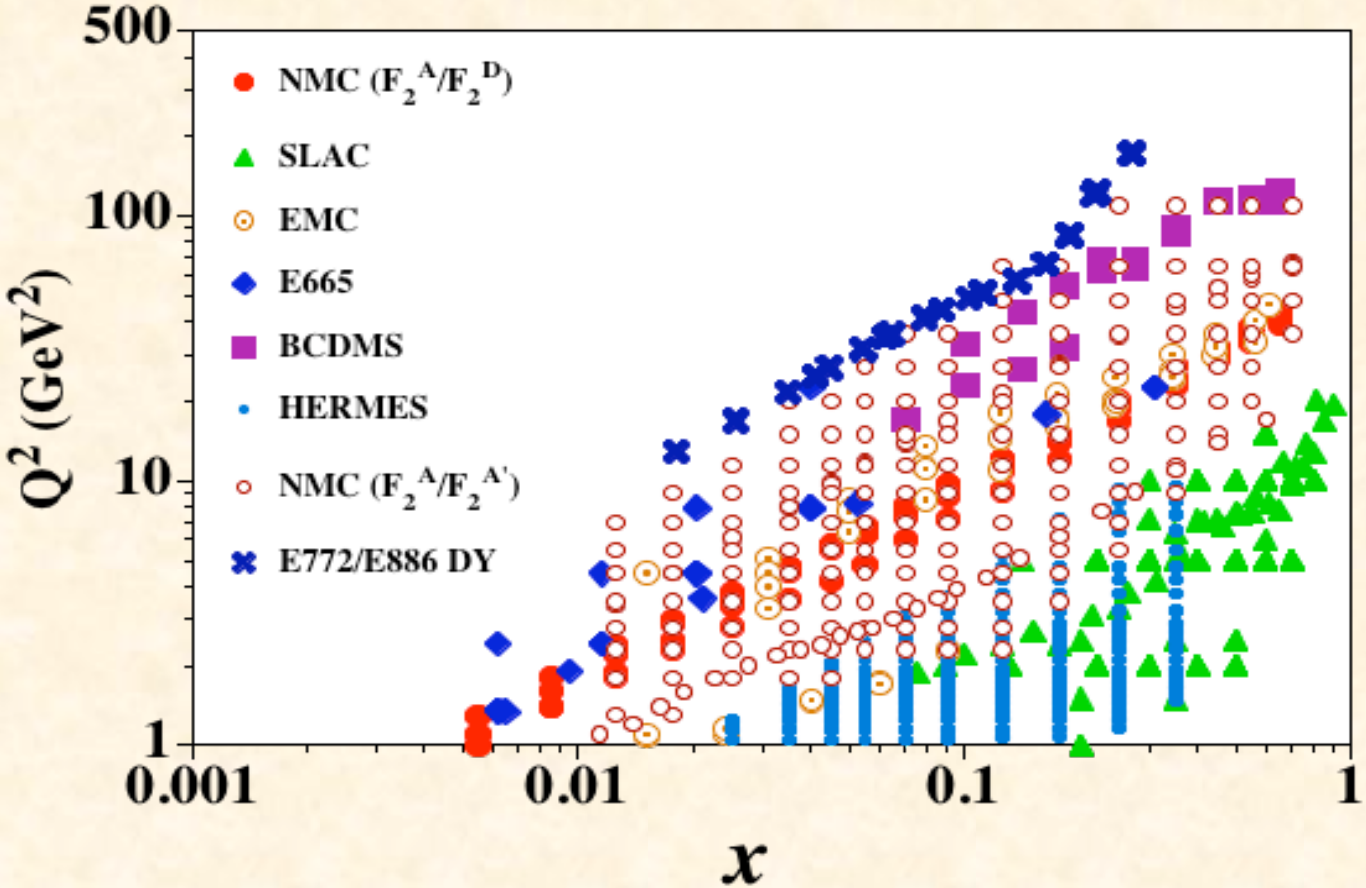
NMC: He, Li, C, Ca
SLAC: He, Be, C, Al, Ca, Fe, Ag, Au
EMC: C, Ca, Cu, Sn
E665: C, Ca, Xe, Pb
BCDMS: N, Fe
HERMES: N, Kr

(2) $F_2^A / F_2^{A'}$ 293 data

NMC: Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$ 52 data

E772: C / D, Ca / D, Fe / D, W / D
E866: Fe / Be, W / Be



Analysis of nuclear F_2 and Drell-Yan data

Initial NPDFs at Q_0^2 : $f_i^A(x,Q_0^2) = w_i(x,A) f_i(x,Q_0^2)$ f_i = nucleonic PDF

Assume $1/A^{1/3}$ dependence for the nuclear modification.

Then, use the functional form:

$$w_i(x,A) = 1 + \left(1 - \frac{1}{A^{1/3}}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

a_i, b_i, c_i, d_i, β_i : parameters to be determined by χ^2 analysis

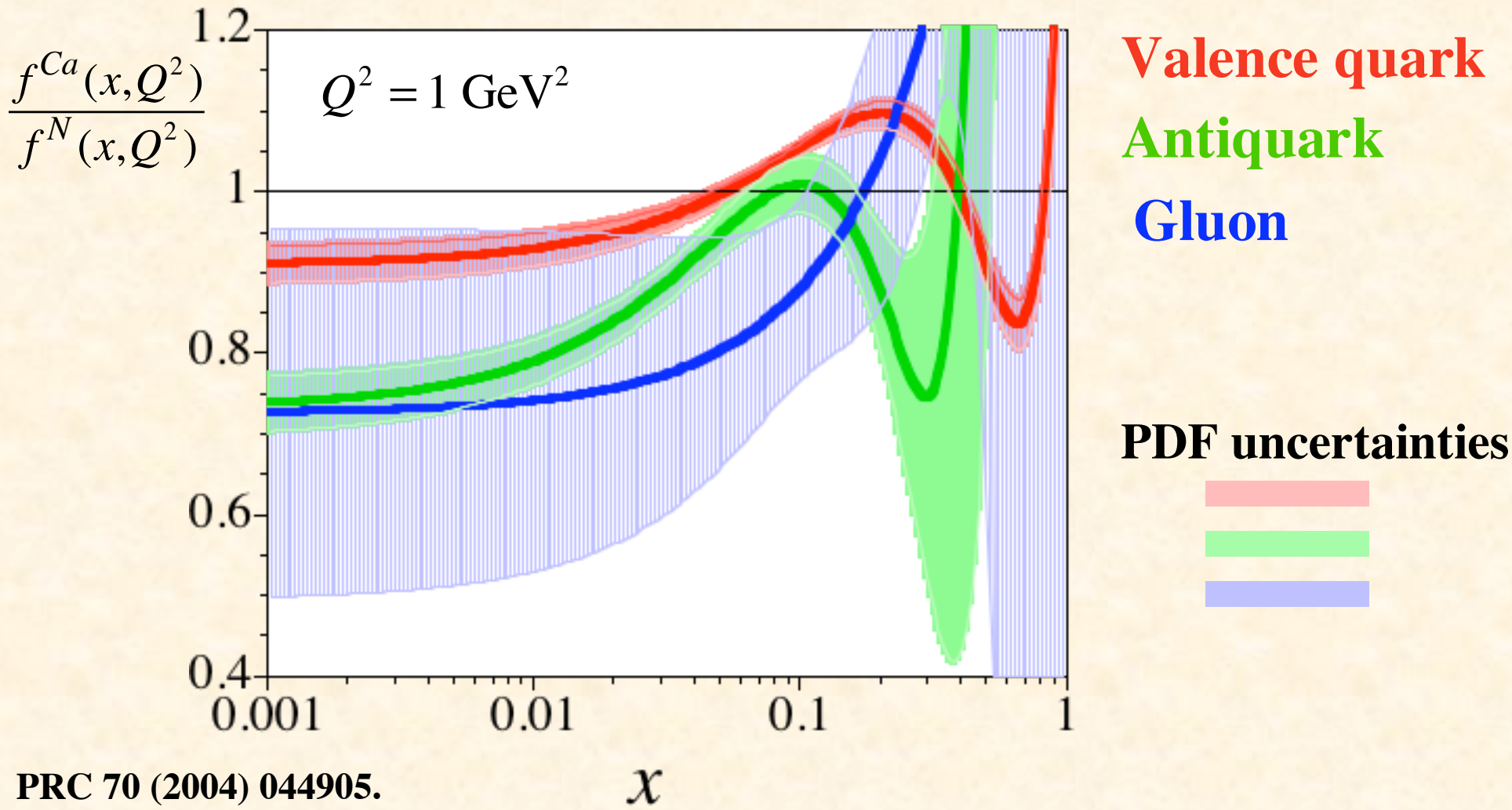
$$\chi^2 = \sum_i \frac{(R_i^{data} - R_i^{calc})^2}{(\sigma_i^{data})^2}, \quad R = \frac{F_2^A}{F_2^D}, \ \frac{F_2^A}{F_2^{A'}}, \ \frac{\sigma_{DY}^{pA}}{\sigma_{DY}^{pD}}, \quad \sigma_i^{data} = \sqrt{(\sigma_i^{syst})^2 + (\sigma_i^{stat})^2}$$

Note: χ^2 is calculated by evolving the NPDFs to experimental Q^2 points by the DGLAP evolution equations.

The error of a distribution $F(x)$ is calculated by the Hessian method:

$$[\delta F(x)]^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F(x)}{\partial \xi_i} H_{ij}^{-1} \frac{\partial F(x)}{\partial \xi_j}, \quad H_{ij} = \text{Hessian}, \quad \xi_i = \text{parameter}$$

Nuclear corrections for ^{40}Ca with uncertainties



PRC 70 (2004) 044905.

PDF-library code can be obtained from
http://research.kek.jp/people/kumanos/nuclp.html

Nuclear-PDF-Modification Effects

on the NuTeV $\sin^2\theta_W$ Determination

Modified Paschos-Wolfenstein relation

$$q_v^A \equiv q^A - \bar{q}^A$$

$$R_A^- = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\bar{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\bar{\nu} A}} = \frac{\{1-(1-y)^2\}\,[\,(u_L^2 - u_R^2)\{u_v^A(x) + c_v^A(x)\} + (d_L^2 - d_R^2)\{d_v^A(x) + s_v^A(x)\}]}{d_v^A(x) + s_v^A(x) - (1-y)^2\{u_v^A(x) + c_v^A(x)\}}$$

(1) Difference between nuclear modifications of u_V and d_V: $\varepsilon_v(x) = \dfrac{w_{d_v}(x) - w_{u_v}(x)}{w_{d_v}(x) + w_{u_v}(x)}$

Nuclear effects are in the weight functions: w_{u_v} and w_{d_v}

$$u_v^A(x) = w_{u_v}(x)\,\frac{Z\,u_v(x) + N\,d_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x)\,\frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

(2) Neutron excess: $\varepsilon_n(x) = \dfrac{N-Z}{A}\,\dfrac{u_V(x) - d_V(x)}{u_V(x) + d_V(x)}$

(3) Strange, Charm: $\varepsilon_s(x),\ \varepsilon_c(x) = \dfrac{2\,s_v^A(x) \quad \text{or} \quad 2\,c_v^A(x)}{[w_{uv}(x) + w_{dv}(x)][u_V(x) + d_V(x)]}$

$$R_A^- = \frac{\begin{array}{c}(\frac{1}{2} - \sin^2\theta_W)\,\{1 + \varepsilon_v(x)\,\varepsilon_n(x)\} + \frac{1}{3}\sin^2\theta_W\,\{\varepsilon_v(x) + \varepsilon_n(x)\} \\ + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\,\varepsilon_s(x) + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)\,\varepsilon_c(x)\end{array}}{1 + \varepsilon_v(x)\,\varepsilon_n(x) + \dfrac{1+(1-y)^2}{1-(1-y)^2}\,\{\varepsilon_v(x) + \varepsilon_n(x)\} + \dfrac{2\{\varepsilon_s(x) - (1-y)^2\,\varepsilon_c(x)\}}{1-(1-y)^2}}$$

Expand in $\varepsilon_v, \varepsilon_n, \varepsilon_s, \varepsilon_c \ll 1$

We investigate this term.

$$\mathbf{R_A^- = \frac{1}{2} - \sin^2\theta_W + O(\varepsilon_v) + O(\varepsilon_n) + O(\varepsilon_s) + O(\varepsilon_c)}$$

Global analysis of F_2 and Drell-Yan data for $\varepsilon_v(x)$

$$u_v^A(x) = w_{u_v}(x,A)\,\frac{Z\,u_v(x) + N\,d_v(x)}{A}$$

$$d_v^A(x) = w_{d_v}(x,A)\,\frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\,\bar{q}(x), \quad g^A(x) = w_g(x,A)\,g(x)$$

in the NPDF analysis

$$w_{uv} = 1 + (1-1/A^{1/3})\,\frac{a_{uv}+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

$$w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a_{dv}+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

in the current analysis

$$w_{uv} + w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a_v+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

$$w_{uv} - w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a'_v+b'_v x+c'_v x^2+d'_v x^3}{(1-x)^{\beta_v}}$$

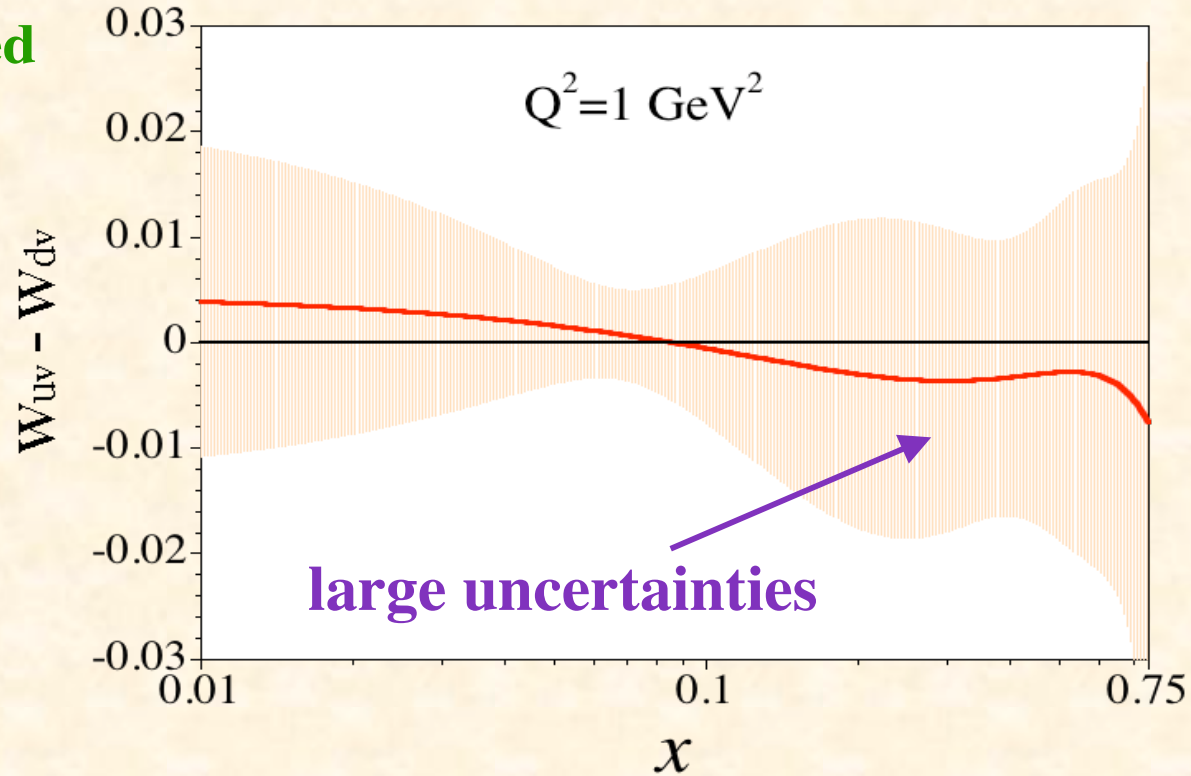
Analysis result for $\varepsilon_v(x)$

$$\varepsilon_v(x) = \frac{w_{d_v}(x) - w_{u_v}(x)}{w_{d_v}(x) + w_{u_v}(x)}$$

$$R_{\bar{A}} = \frac{1}{2} - \sin^2\theta_W - \varepsilon_v(x)\left\{\left(\frac{1}{2} - \sin^2\theta_W\right)\frac{1+(1-y)^2}{1-(1-y)^2} - \frac{1}{3}\sin^2\theta_W\right\} + O(\varepsilon_v^2)$$

$$w_{uv} - w_{dv} = 1 + (1 - 1/A^{1/3})\,\frac{a'_v + b'_v x + c'_v x^2 + d'_v x^3}{(1-x)^{\beta_v}}$$

a'_v, b'_v, c'_v, d'_v are determined by the analysis



PRD 71 (2005) 113007

$\varepsilon_v(x)$ effects on $\sin^2\theta_W$

2002 version

PRD 66 (2002) 111301

Constraints of baryon number and charge

$$Z = \int dx\, A \sum_q e_q (q^A - \bar{q}^A) = \int dx\, \frac{A}{3} (2\, u_V^A - d_V^A)$$

$$A = \int dx\, A \sum_q \frac{1}{3} (q^A - \bar{q}^A) = \int dx\, \frac{A}{3} (u_V^A + d_V^A)$$

$$\longrightarrow \quad (A) \quad \int dx\, (u_v + d_v)\, [\, \Delta w_v + w_v\, \varepsilon_v(x)\, \varepsilon_n(x)\,] = 0$$

$$(B) \quad \int dx\, (u_v + d_v)\, [\, \Delta w_v\, \{1 - 3\, \varepsilon_n(x)\} - w_v\, \varepsilon_v(x)\, \{3 - \varepsilon_n(x)\}\,] = 0$$

$$\text{where} \quad w_v = \frac{w_{u_v} + w_{d_v}}{2}, \quad \Delta w_v = w_v - 1$$

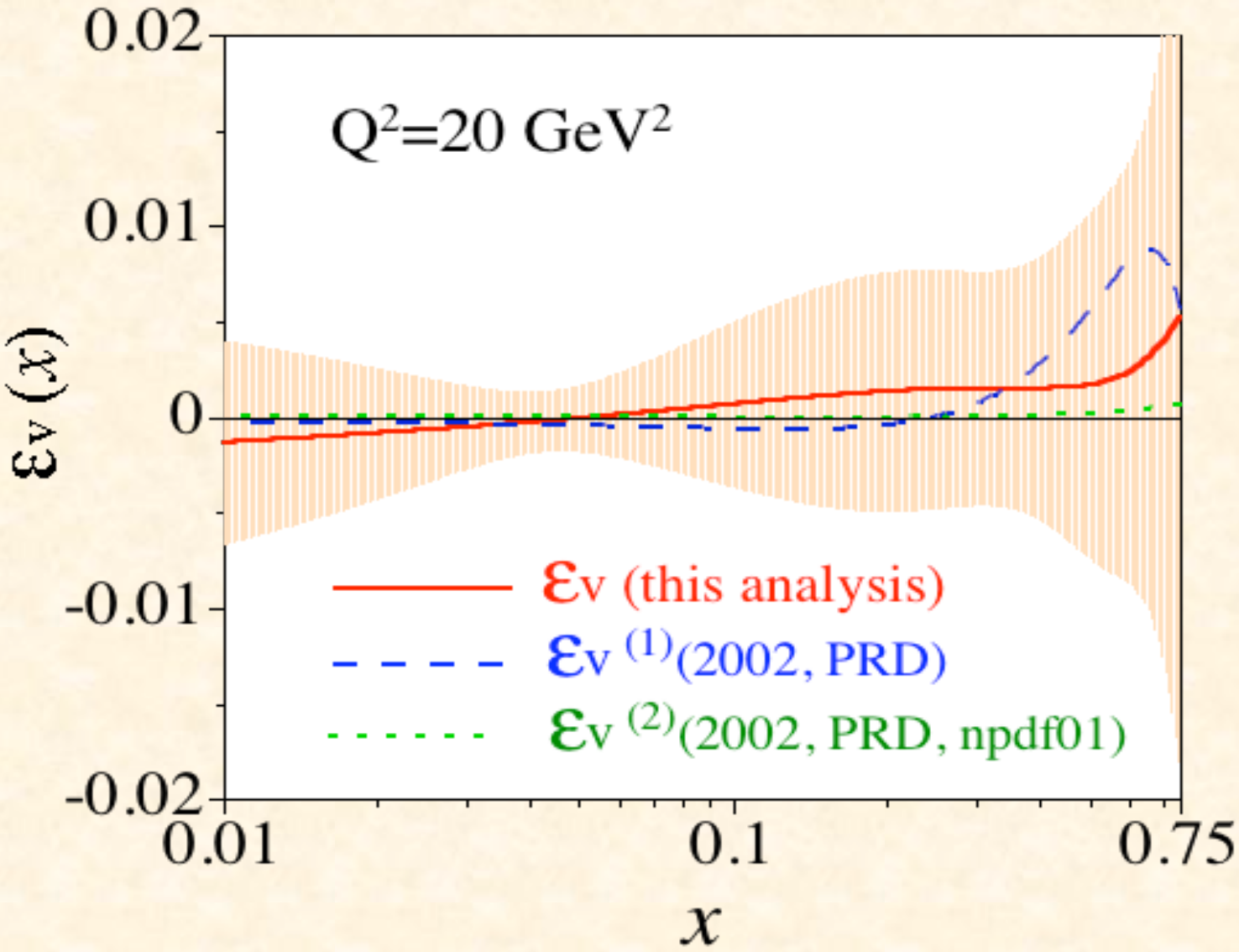
Prescription 1. Neglect $O(\varepsilon^2)$, then integrand (B) = 0

$$\varepsilon_V^{(1)}(x) = -\frac{N - Z}{A} \frac{u_V(x) - d_V(x)}{u_V(x) + d_V(x)} \frac{\Delta w_V(x)}{w_V(x)}$$

Prescription 2. χ^2 analysis of NPDFs

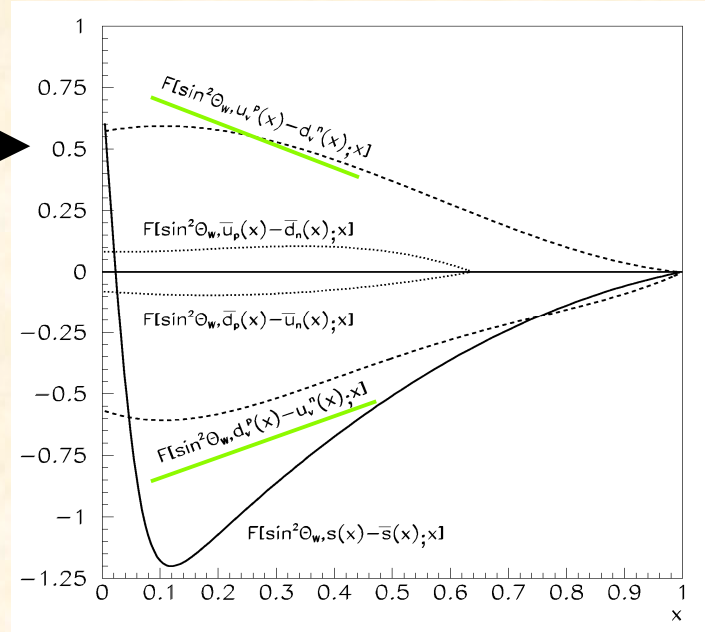
$$\varepsilon_V^{(2)}(x) = \frac{w_{d_v}(x) - w_{u_v}(x)}{w_{d_v}(x) + w_{u_v}(x)}$$

Comparison with the 2002 results



NuTeV kinematics

G. P. Zeller et al. Phys. Rev. D65 (2002) 111103.



x

PDFs $\leftrightarrow$ NuTeV PDFs (*)

$$xu_V^A = w_{u_v} \frac{Z\, xu_v + N\, xd_v}{A} = \frac{Z\, u^*_{vp} + N\, u^*_{vn}}{A}$$

$$xd_V^A = w_{d_v} \frac{Z\, xd_v + N\, xu_v}{A} = \frac{Z\, d^*_{vp} + N\, d^*_{vn}}{A}$$

$\rightarrow$ $u^*_{vp}=w_{u_v}xu_v$, $u^*_{vn}=w_{u_v}xd_v$, $d^*_{vp}=w_{d_v}xd_v$, $d^*_{vn}=w_{d_v}xu_v$

$$\rightarrow \delta u^*_v = u^*_{vp} - d^*_{vn} = -\,\varepsilon_v\,(w_{u_v} + w_{d_v})\, xu_v$$

$$\delta d^*_v = d^*_{vp} - u^*_{vn} = +\,\varepsilon_v\,(w_{u_v} + w_{d_v})\, xd_v$$

$$\Delta \sin^2\theta_W = -\int dx\, \{ \underline{F\,[\delta u^*_v, x]}\, \delta u^*_v + \underline{F\,[\delta d^*_v, x]}\, \delta d^*_v \}$$

$$= 0.0004 \pm 0.0015$$

at Q^2=20 GeV^2

Summary on NuTeV $\sin^2\theta_W$

(1) Nuclear modifications could be important for understanding the NuTeV anomaly. We provide nuclear PDFs for general users at http://research.kek.jp/people/kumanos/nuclp.html.

(2) χ^2 analysis for the difference between nuclear modifications of u_v and d_v distributions.

It is very difficult to determine it at this stage.

(3) Effect on NuTeV $\sin^2\theta_W$

$\Delta(\sin^2\theta_W) = 0.0004 \pm 0.0015$ (with a large error)

Note that there is a possibility of error underestimation.

Further studies are needed for the nuclear modification.