Chiral dynamics in the $\gamma p \to \pi^0 \eta p$ and $\gamma p \to \pi^0 K^0 \Sigma^+$ reactions

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Affiliation

CSIC

EtaMesonNet

arXiv:nucl-th/0510015, nucl-th/0510057

Outline

- Forthcoming experiments at ELSA: Prediction of invariant mass spectra and cross sections for the reactions

$$\gamma p \to \pi^0 \eta p \quad \text{and} \quad \gamma p \to \pi^0 K^0 \Sigma^+$$

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  - The role of dynamically generated $3/2^-$ resonances from 0^- meson octet $\otimes$ $3/2^+$ decuplet interaction

Dynamically generated $N^*(1535)$

N. Kaiser et. al., PLB **362** (1995) 23, NPA **612** (1997) 297,
E. Oset *et. al.*, NPA **635** (1998) 99
T. Inoue *et. al.*, PRC **65** (2002) 035204

- Coupled channels: $\pi^0 p,\ \pi^+ n,\ \eta p,\ K^+\Sigma^0,\ K^+\Lambda,\ K^0\Sigma^+$

$$\begin{aligned}\mathcal{L}^{(1)}_{\pi N} &= \left\langle \bar{B} i\gamma^\mu \nabla_\mu B\right\rangle - M_B \left\langle \bar{B} B\right\rangle \\ &\quad + \frac{1}{2} D \left\langle \bar{B}\gamma^\mu\gamma_5 \left\{u_\mu, B\right\}\right\rangle + \frac{1}{2} F \left\langle \bar{B}\gamma^\mu\gamma_5 [u_\mu, B]\right\rangle\end{aligned}$$

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- Kernel:

$$V_{ij} = -C_{ij}\ \frac{1}{4 f_i f_j}\ \overline{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu)$$

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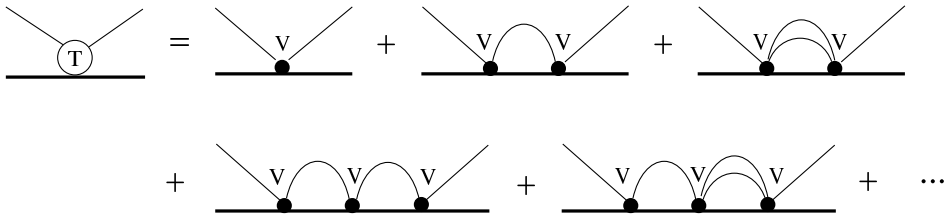
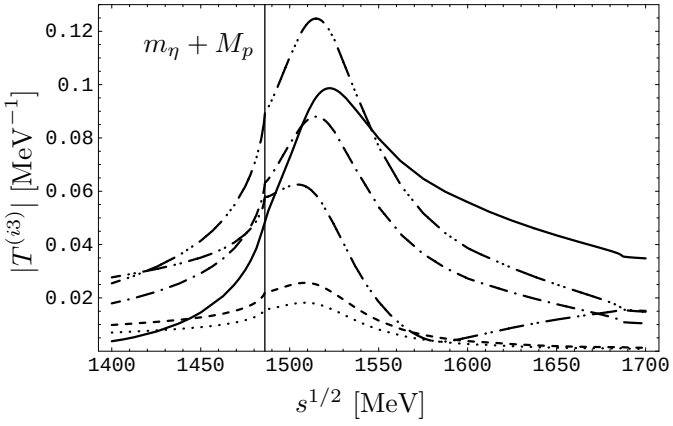
- Bethe–Salpeter Equation:

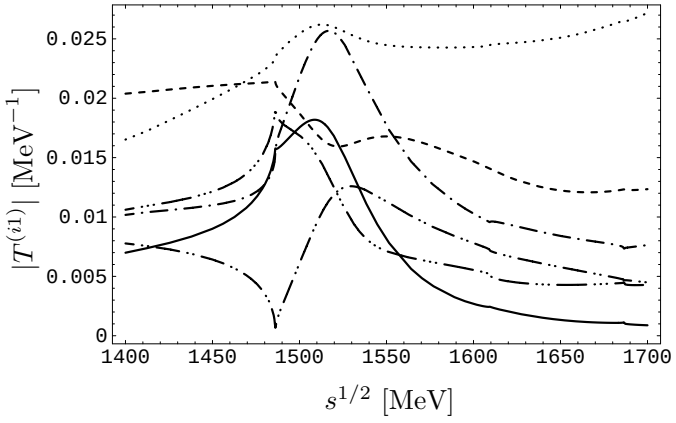
$$T = V + VGT,$$



Amplitudes and partial waves

- $M_i B_i \to \eta p$ and $M_i B_i \to \pi^0 p$ transition amplitudes





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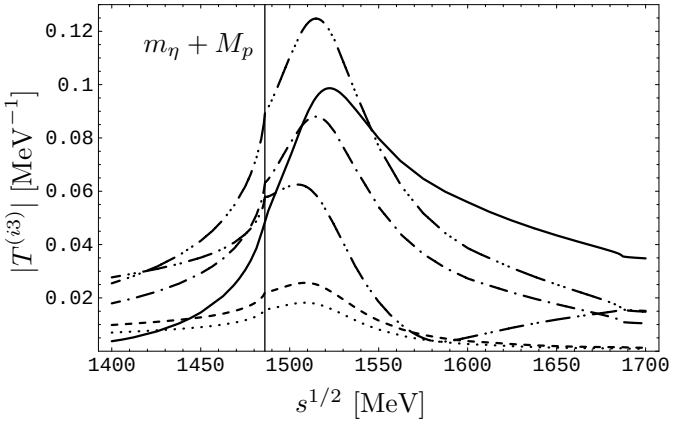
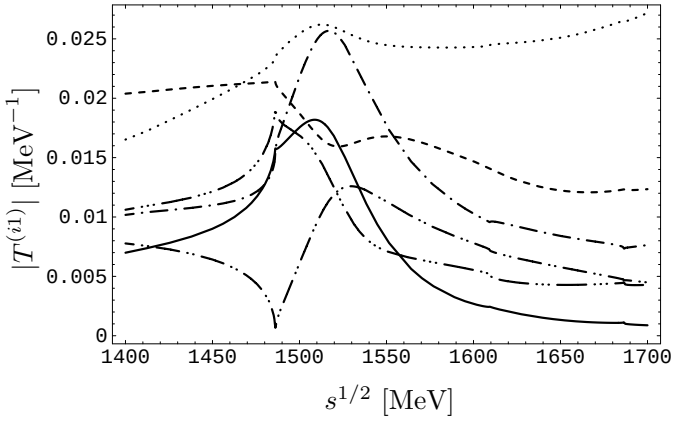
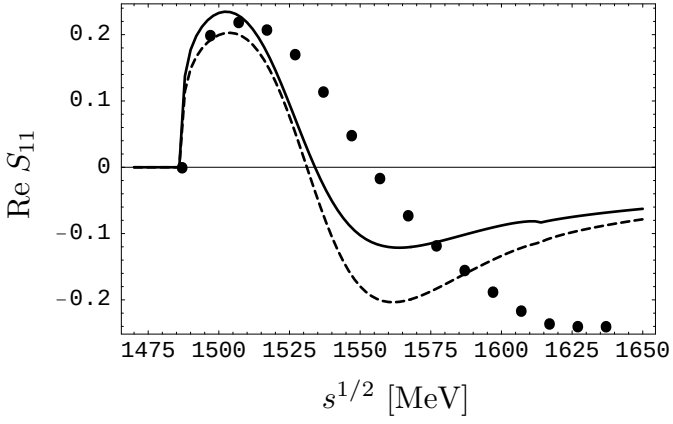
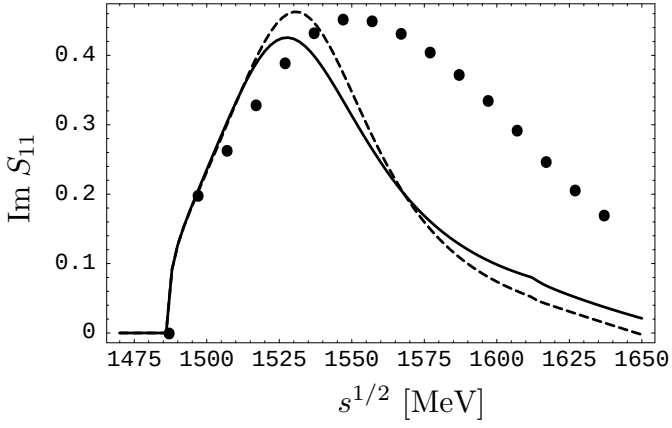




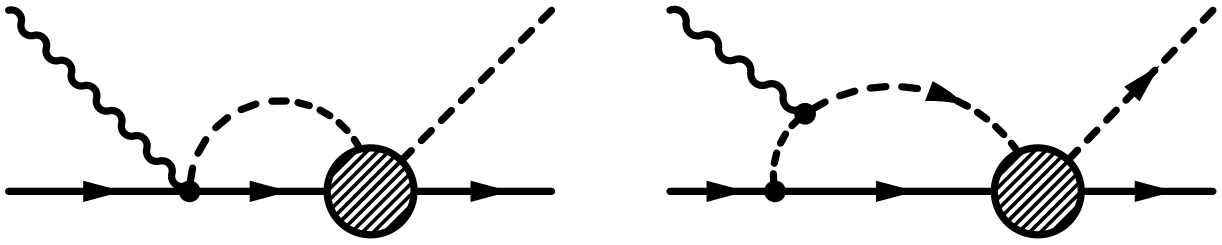
- S_{11} partial wave in $\pi N \to \eta N$





The $N^*(1535)$ in $\gamma p \to \eta p$

- Kroll-Ruderman and meson pole term:



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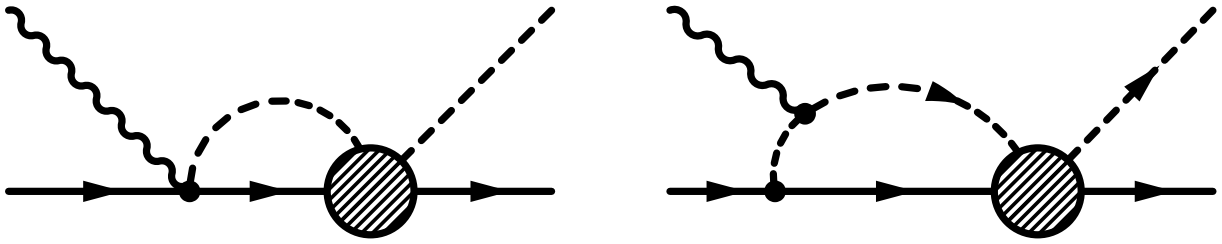
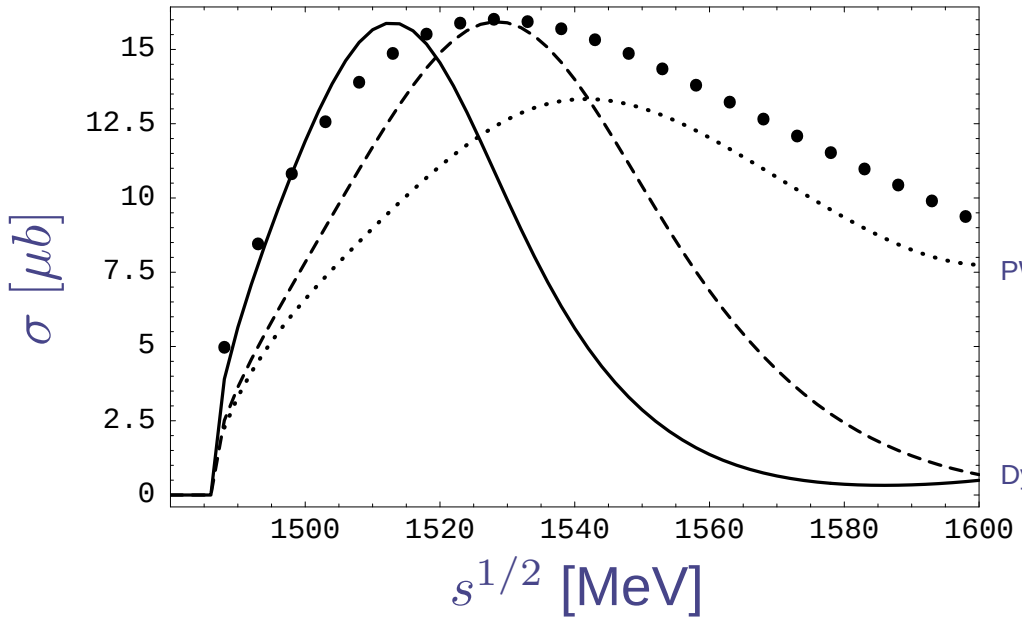


- Cross section for η photoproduction



PWA from R.A. Arndt *et.al.* PRC **66**, 035213 (2004)

Dynamically generated $N^*(1535)$

$\gamma p \to \pi^0 \eta p$: Nucleon magnetic momentum

- Effective Lagrangian Meissner *et.al.*, NPB **499** (1997), 349

$$
\begin{aligned}
\mathcal{L} &= -\frac{i}{4M_p}\, b_6^F \left\langle \overline{B}\,[S^\mu, S^\nu]\,\left[F^+_{\mu\nu}, B\right]\right\rangle \\
&\quad -\frac{i}{4M_p}\, b_6^D \left\langle \overline{B}\,[S^\mu, S^\nu]\,\{F^+_{\mu\nu}, B\}\right\rangle \\
F^+_{\mu\nu} &= -e\left(u^\dagger Q F_{\mu\nu}\, u + u Q F_{\mu\nu}\, u^\dagger\right), \\
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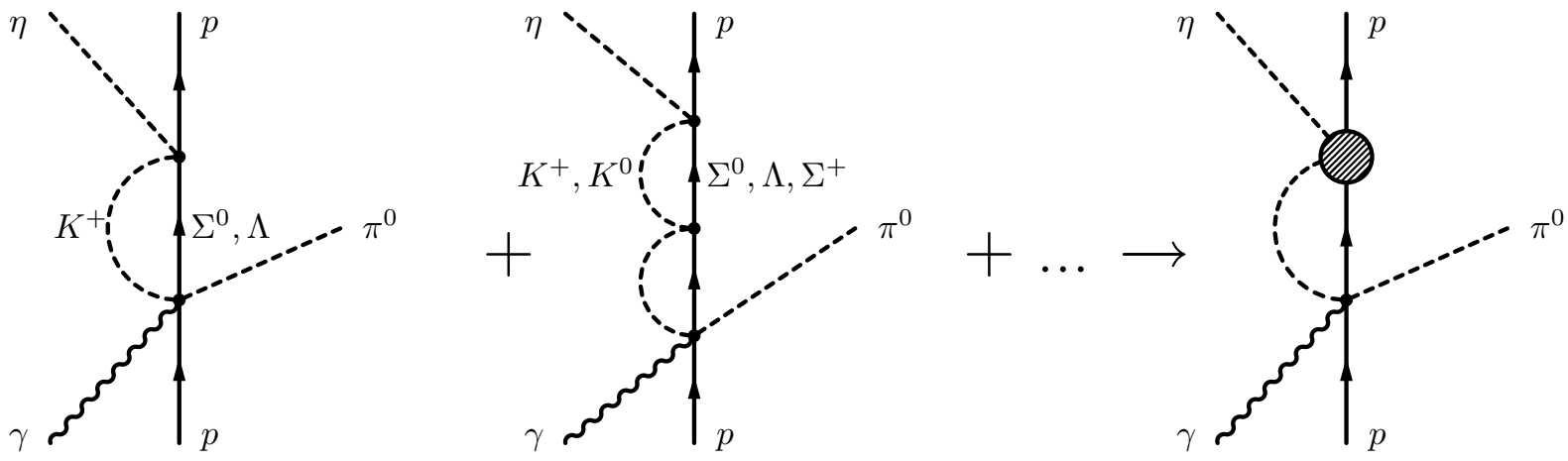
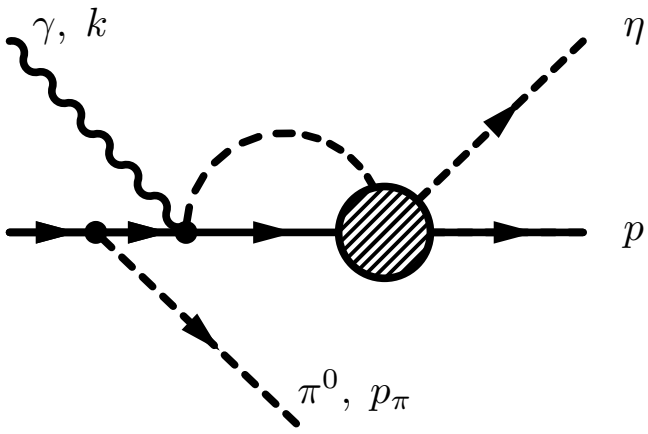
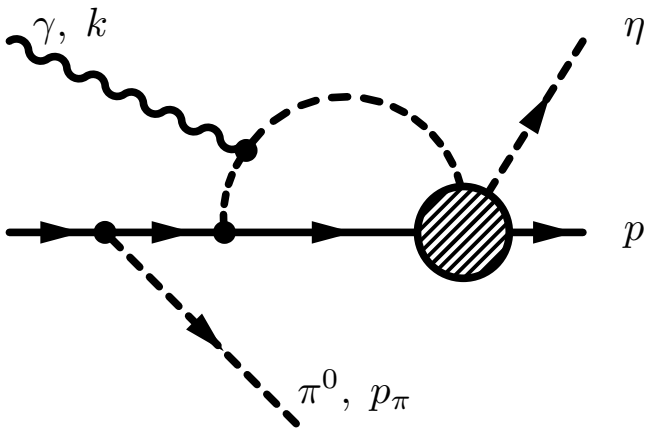
- Unitarization:



Processes with chiral MB interactions

- Kroll-Ruderman and meson pole term as sub-processes in $\pi^0\eta$ production:





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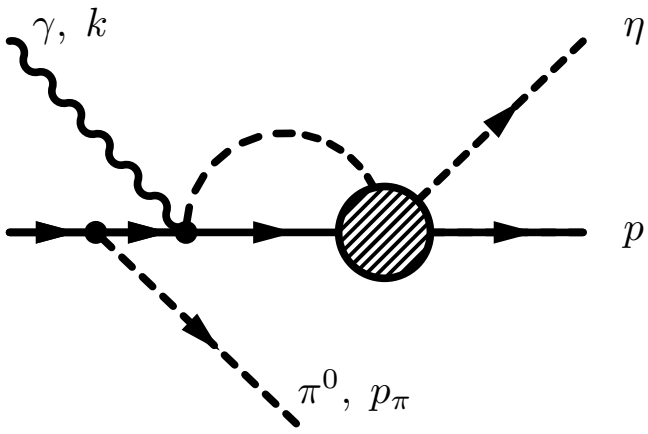
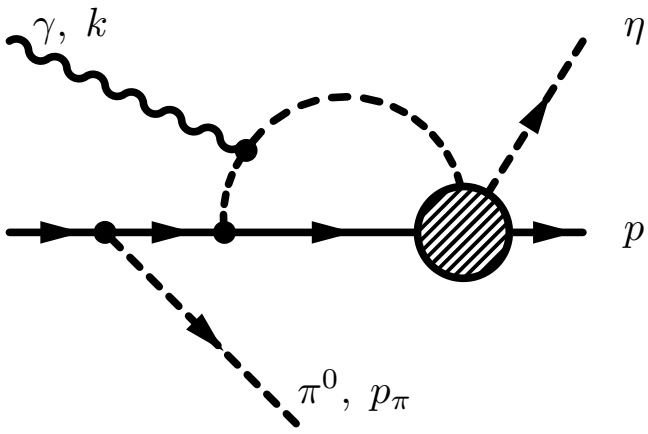
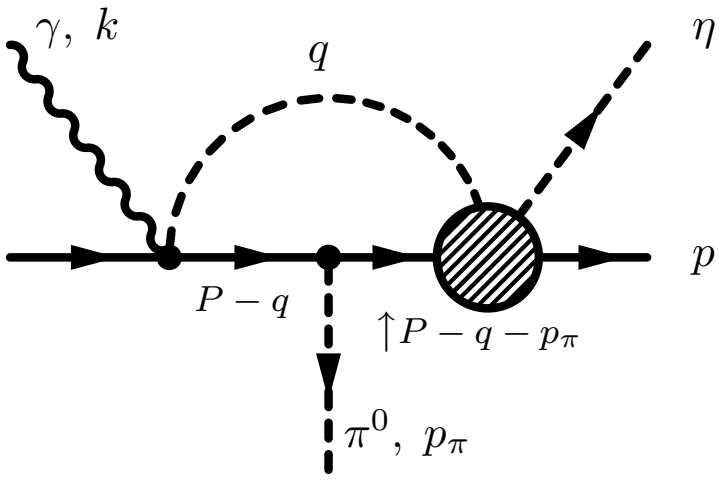
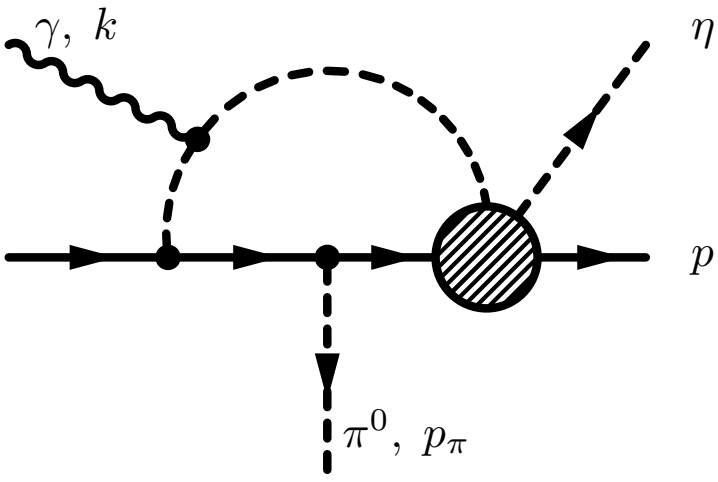




- Pion emission from inside first loop:





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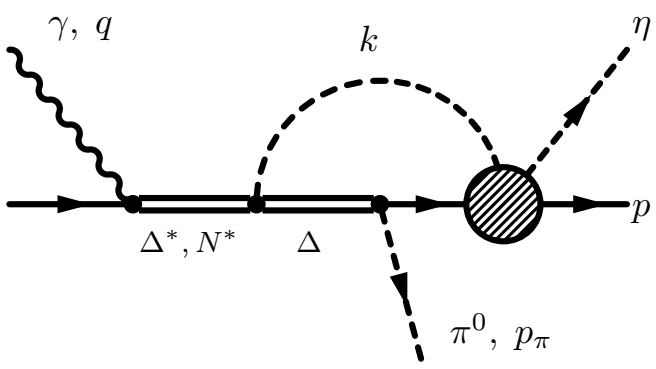

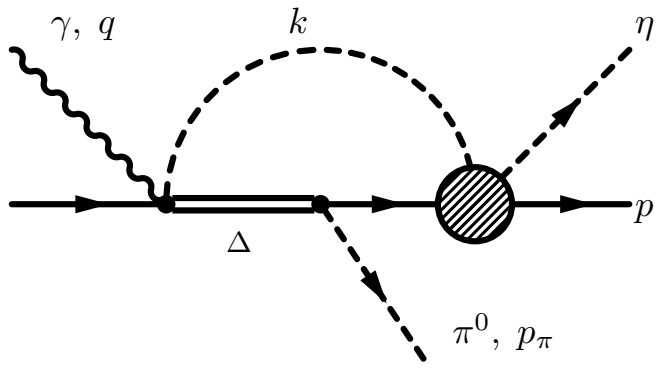

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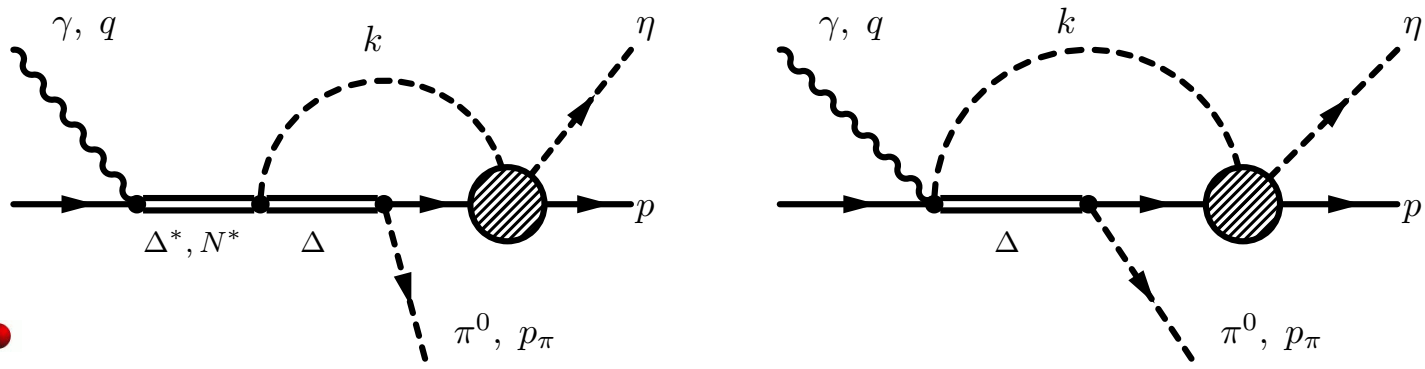

- $$T_{\gamma p\to\pi^0\eta p} = \sum_{i=1,2} T^{(i,3)}(z)\frac{1}{(2\pi)^2}\int_0^\Lambda dq\int_{-1}^{1} dx\ t^i_\Delta \frac{q^2}{2\omega}\frac{M}{E}\frac{1}{\sqrt{s}-\omega-p^0_\pi-E+i\epsilon}\ \frac{1}{\sqrt{s_\Delta}-M_\Delta+i\ \frac{\Gamma(\sqrt{s_\Delta})}{2}}$$

$SU(3)$ coupling of dynamically generated Δ^*

- 0^- meson octet $\otimes$ $3/2^+$ baryon decuplet $\longrightarrow$ $3/2^-$ resonances
  J.A. Oller *et.al.*, PLB **500** (2001) 263,
  M.F.M Lutz *et.al.*, NPA **700** (2002) 193, NPA **706** (2002) 431
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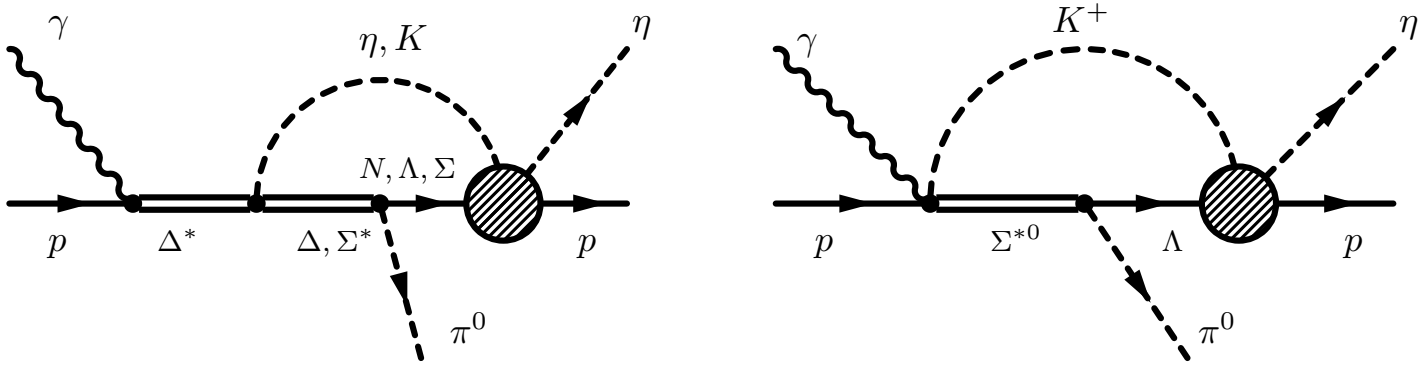

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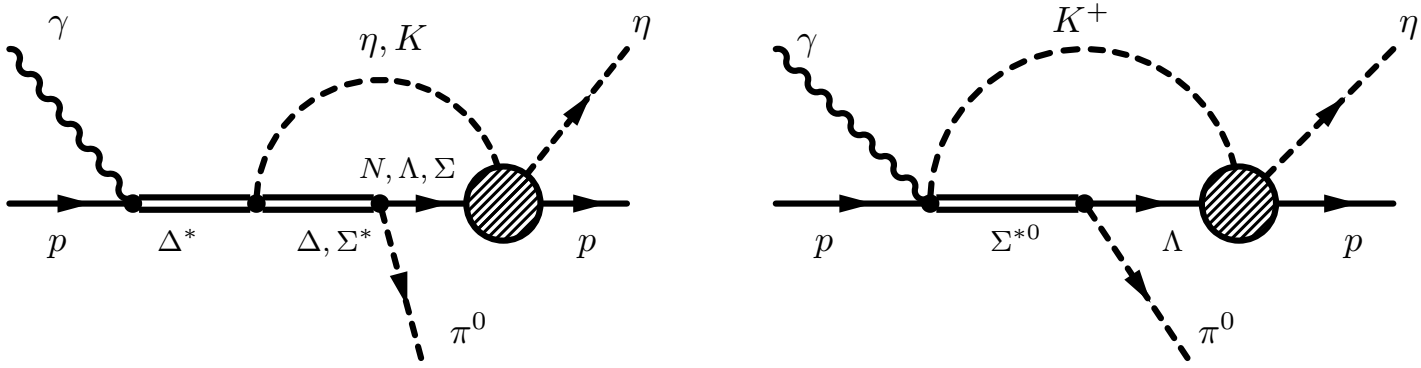


$$t^{(3)}_{\eta\Delta^+p} = -\sqrt{\frac{2}{3}}\, g_\eta\, \frac{f^*_{\Delta N\pi}}{m_\pi}\, G_{\Delta^*}(\sqrt{s})\, \vec{S}\cdot\mathbf{p}_\pi \left[-ig_1'\, \frac{\vec{S}^\dagger\cdot\mathbf{k}}{2M}\,(\vec{\sigma}\times\mathbf{k})\vec{\epsilon} - \vec{S}^\dagger\cdot\vec{\epsilon}\left(g_1'(k^0+\frac{\mathbf{k}^2}{2M}) + g_2'\sqrt{s}\,k^0\right)\right],$$

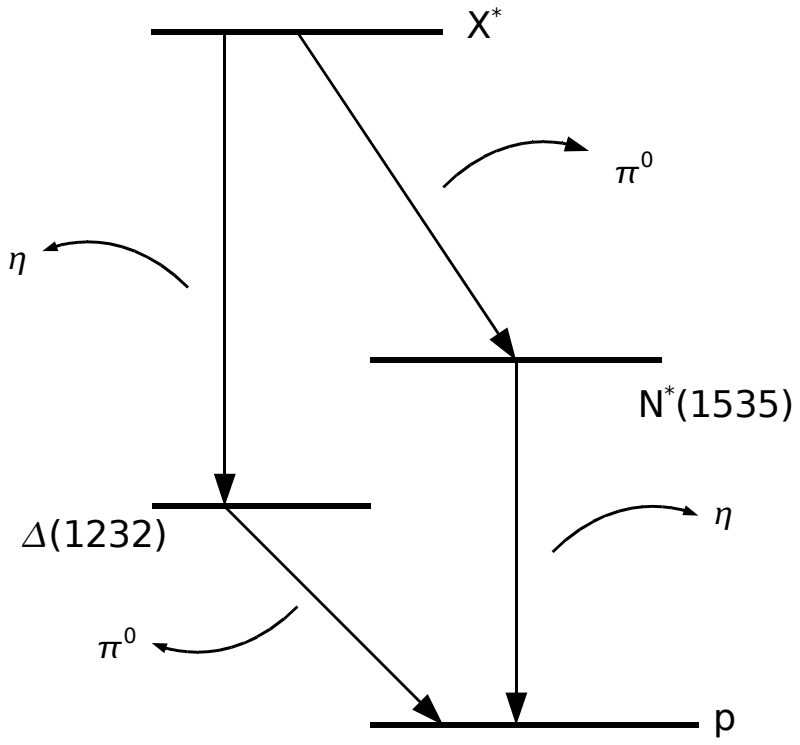
$$t^{(5)}_{K^+\Sigma^{*0}\Lambda} = 1.15\sqrt{\frac{24}{25}}\, g_K\, \frac{D+F}{2f_\pi}\, G_{\Delta^*}(\sqrt{s})\vec{S}\cdot\mathbf{p}_\pi \left[-ig_1'\frac{\vec{S}^\dagger\cdot\mathbf{k}}{2M}(\vec{\sigma}\times\mathbf{k})\cdot\vec{\epsilon} - \vec{S}^\dagger\cdot\vec{\epsilon}\left(g_1'(k^0+\frac{\mathbf{k}^2}{2M}) + g_2'\sqrt{s}\,k^0\right)\right],$$

$$t^{(6)}_{K^0\Sigma^{*+}\Sigma^+} = -\frac{2}{5}\frac{D+F}{2f_\pi}\, g_K\, G_{\Delta^*}(\sqrt{s})\vec{S}\cdot\mathbf{p}_\pi \left[-ig_1'\frac{\vec{S}^\dagger\cdot\mathbf{k}}{2M}(\vec{\sigma}\times\mathbf{k})\cdot\vec{\epsilon} - \vec{S}^\dagger\cdot\vec{\epsilon}\left(g_1'(k^0+\frac{\mathbf{k}^2}{2M}) + g_2'\sqrt{s}\,k^0\right)\right],$$

$$t^{(5)}_{\Sigma^*-\mathrm{KR}} = 1.15\, e\, \frac{4\sqrt{3}}{25}\left(\frac{D+F}{2f_\pi}\right)^2 (2\mathbf{p}_\pi - i(\vec{\sigma}\times\mathbf{p}_\pi))\cdot\vec{\epsilon}$$

Background term

- De-excitation modes into $\pi^0 \eta p$ final state:



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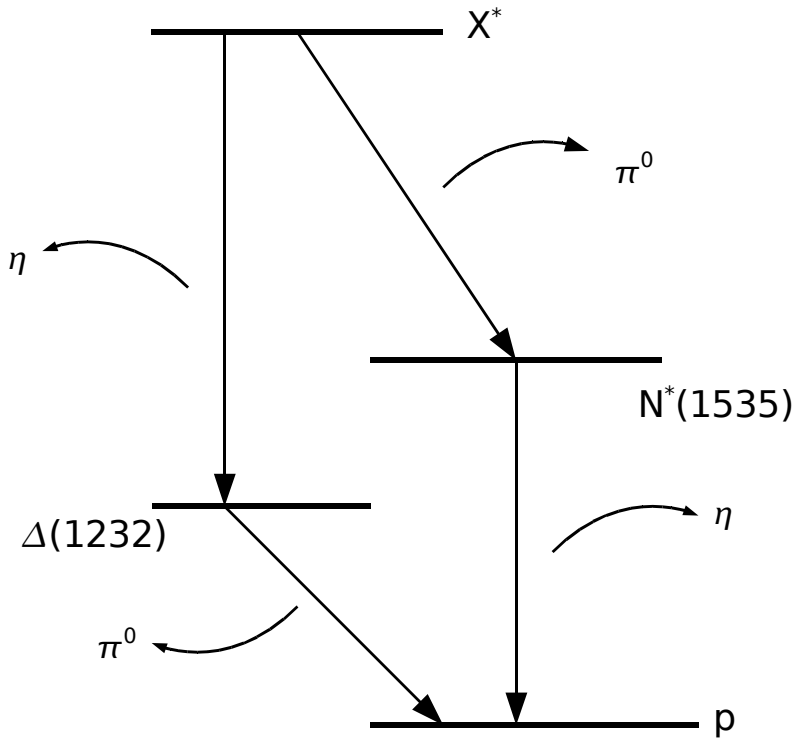

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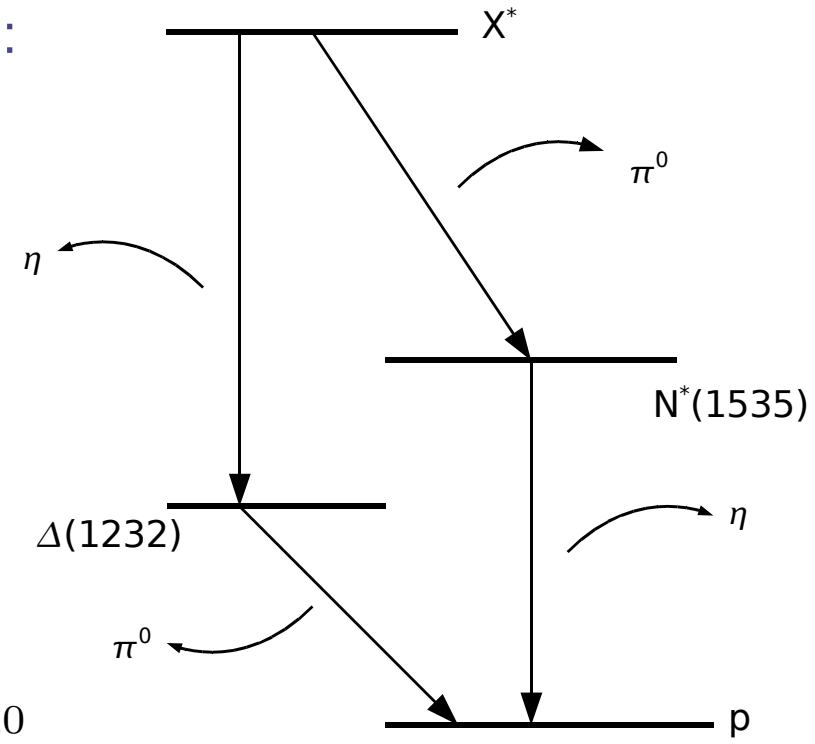

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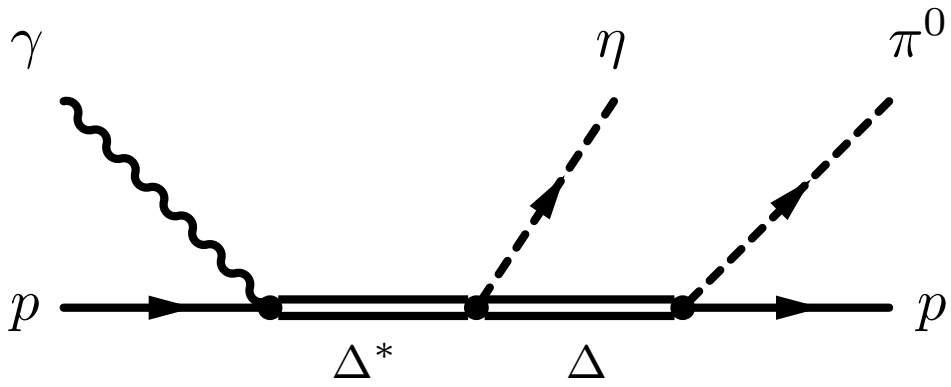
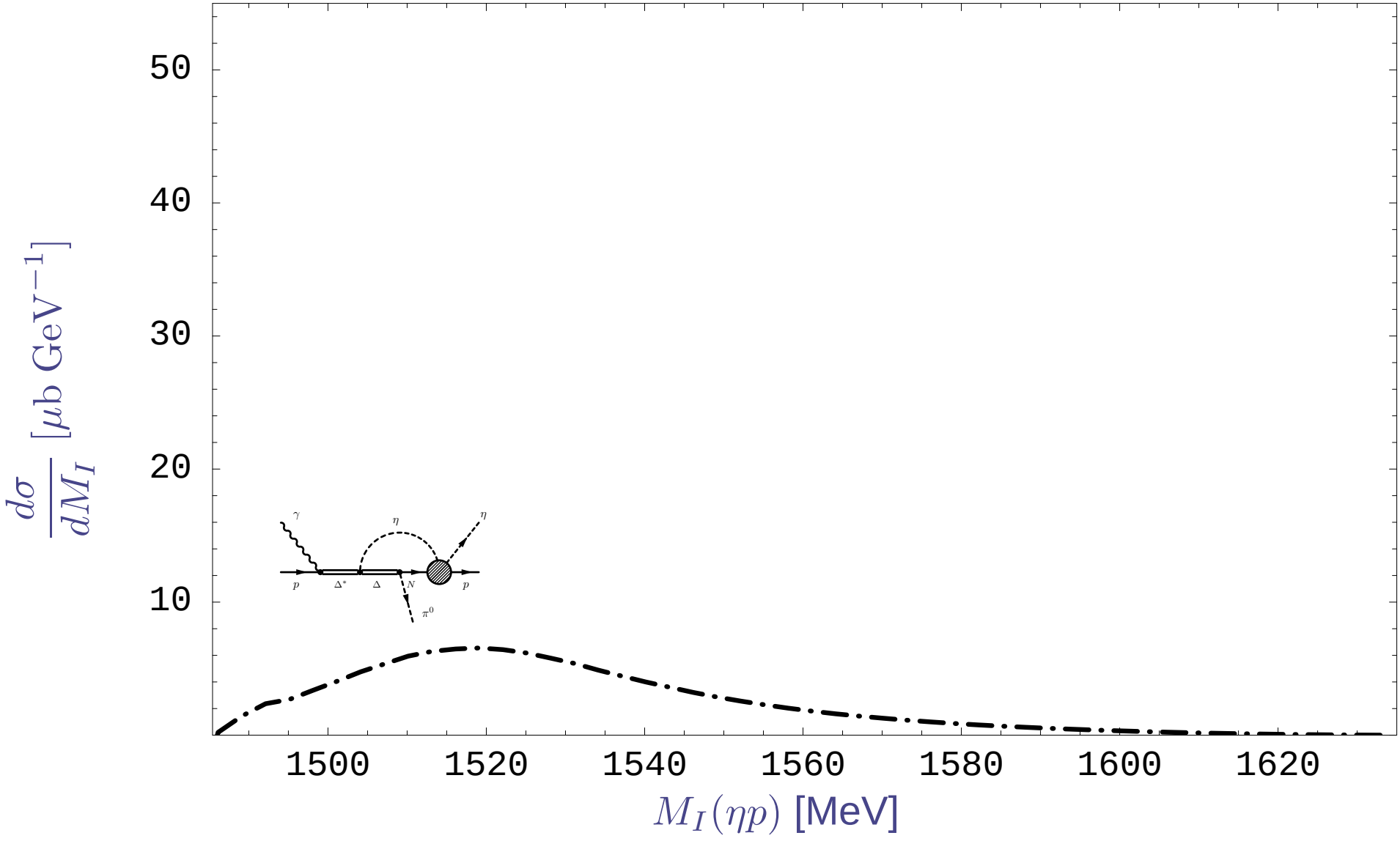
- Tree level contribution:



Invariant mass $M_I(\eta p)$ for $\gamma p \to \pi^0 \eta p$



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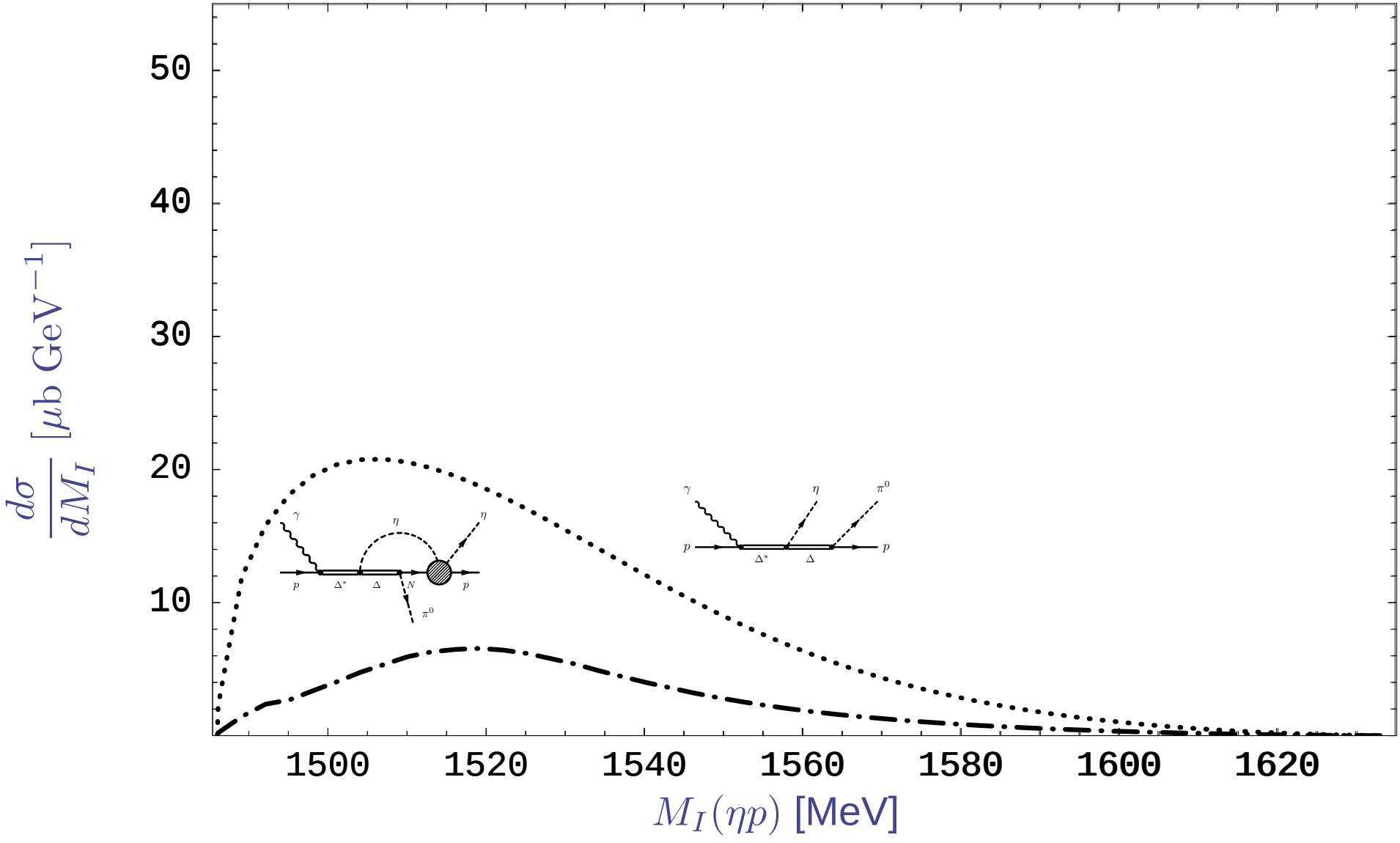

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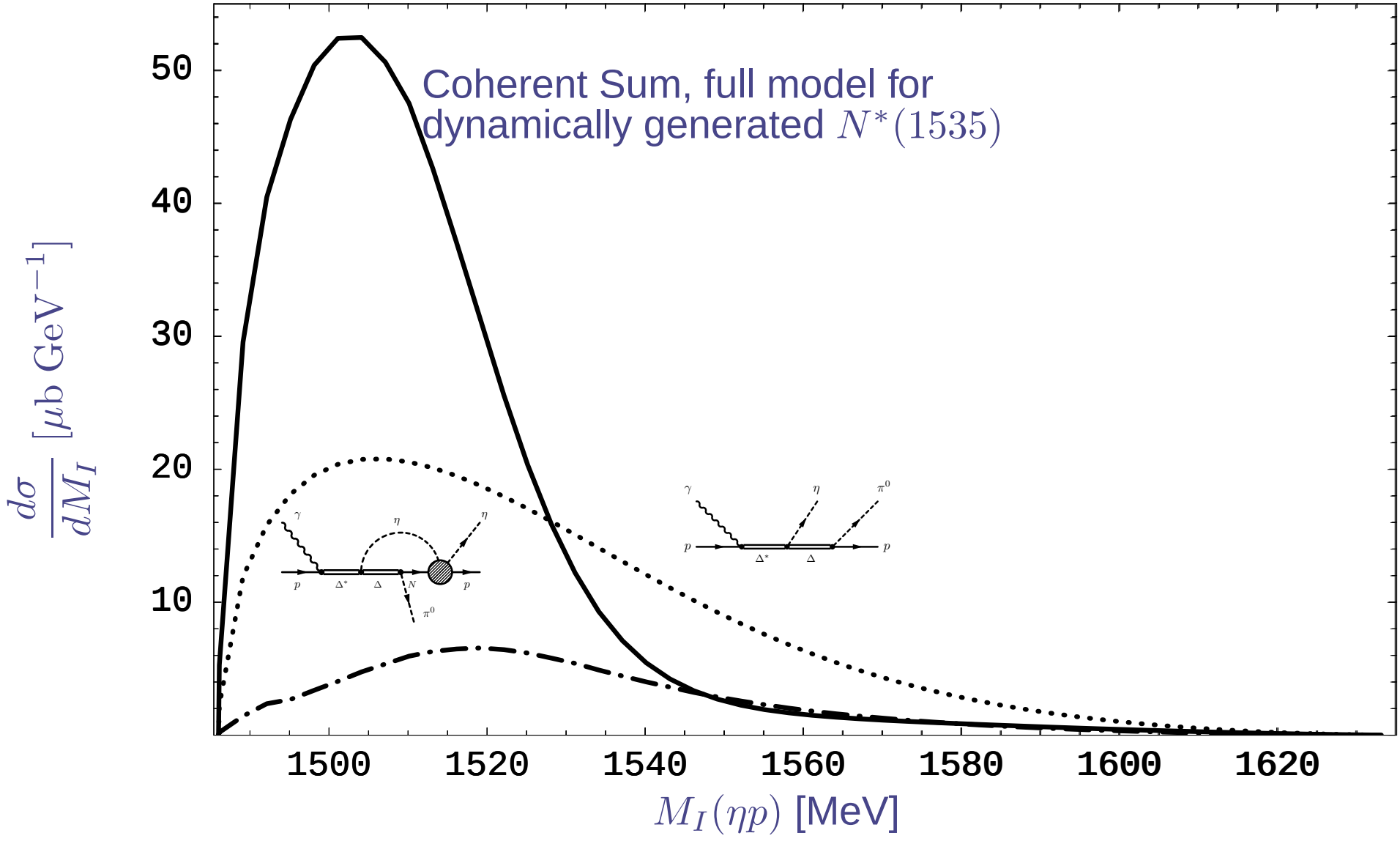

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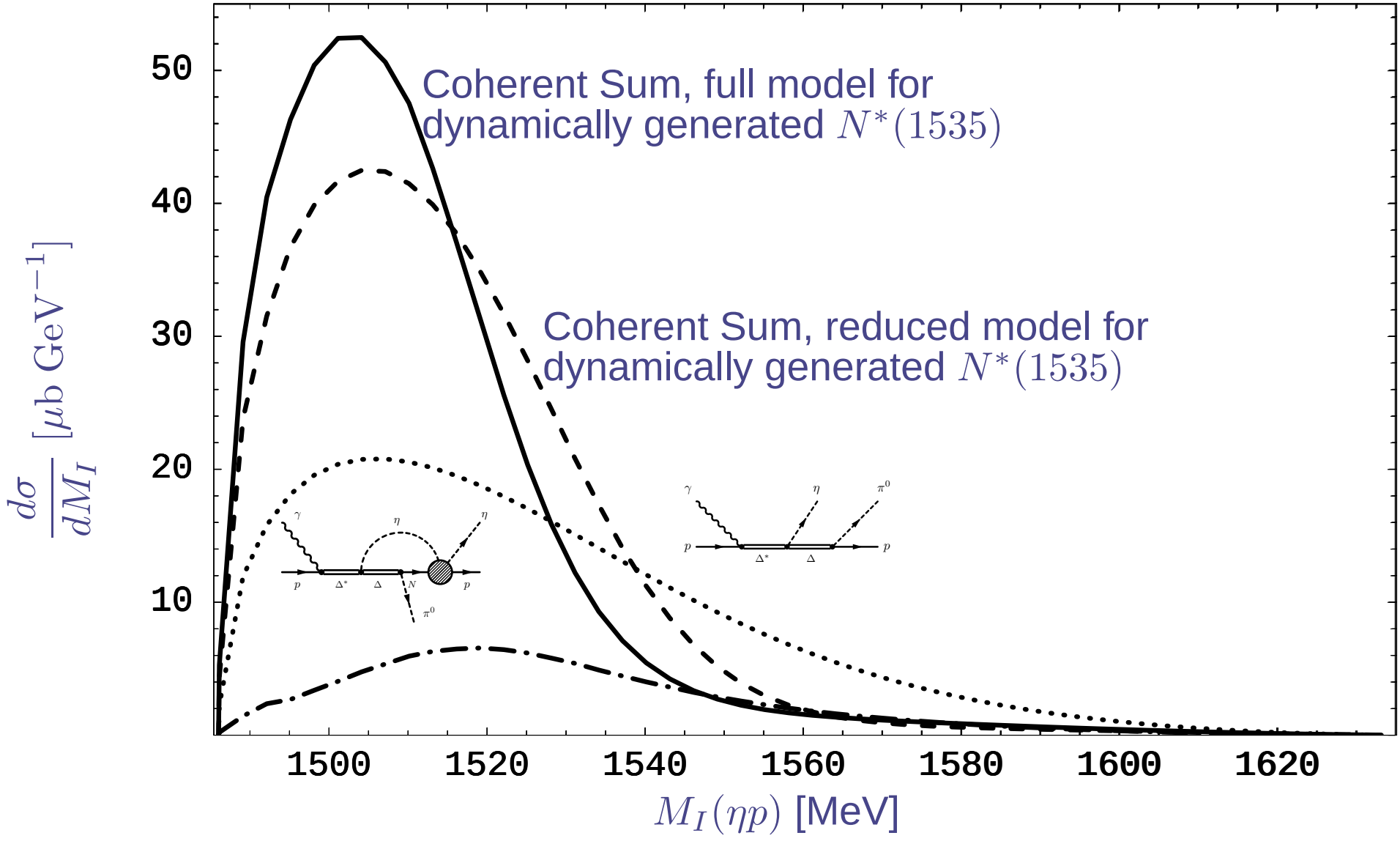
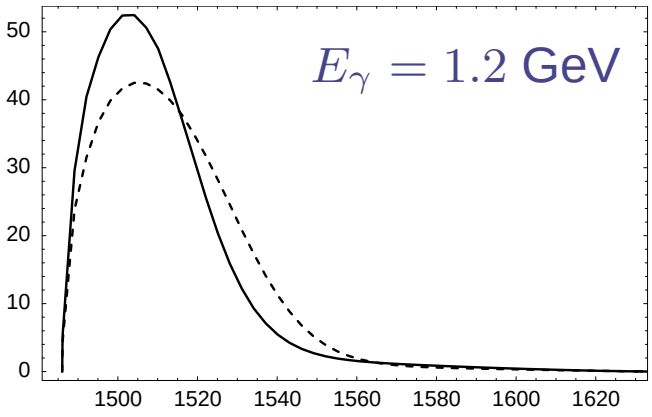


Invariant mass at higher E_γ



Invariant mass spectrum $\frac{d\sigma}{dM_I(\eta p)}$ $[\mu\mathrm{b}\ \mathrm{GeV}^{-1}]$ as a function of M_I [MeV] for various photon lab energies E_γ. Solid and dashed lines: Full and reduced model for the $N^*(1535)$, respectively.

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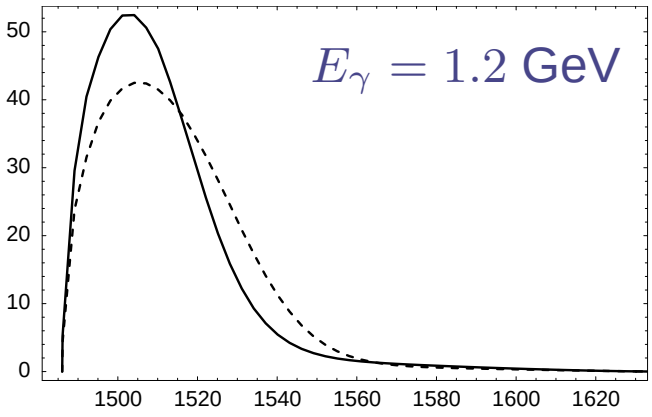

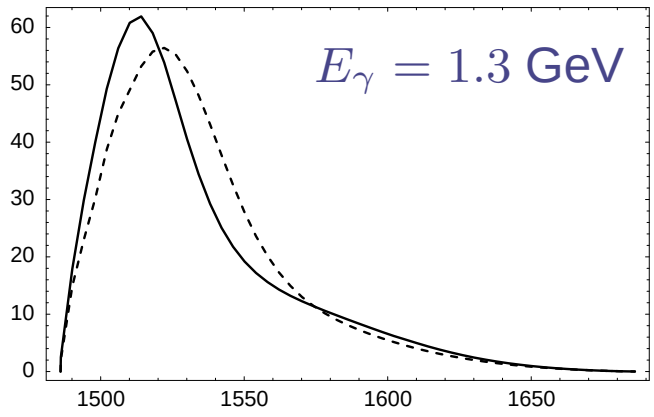

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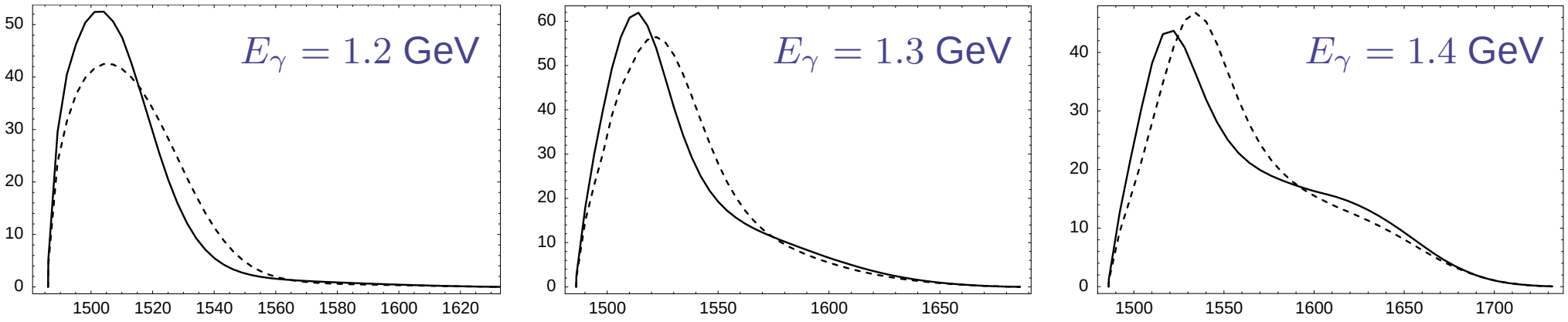

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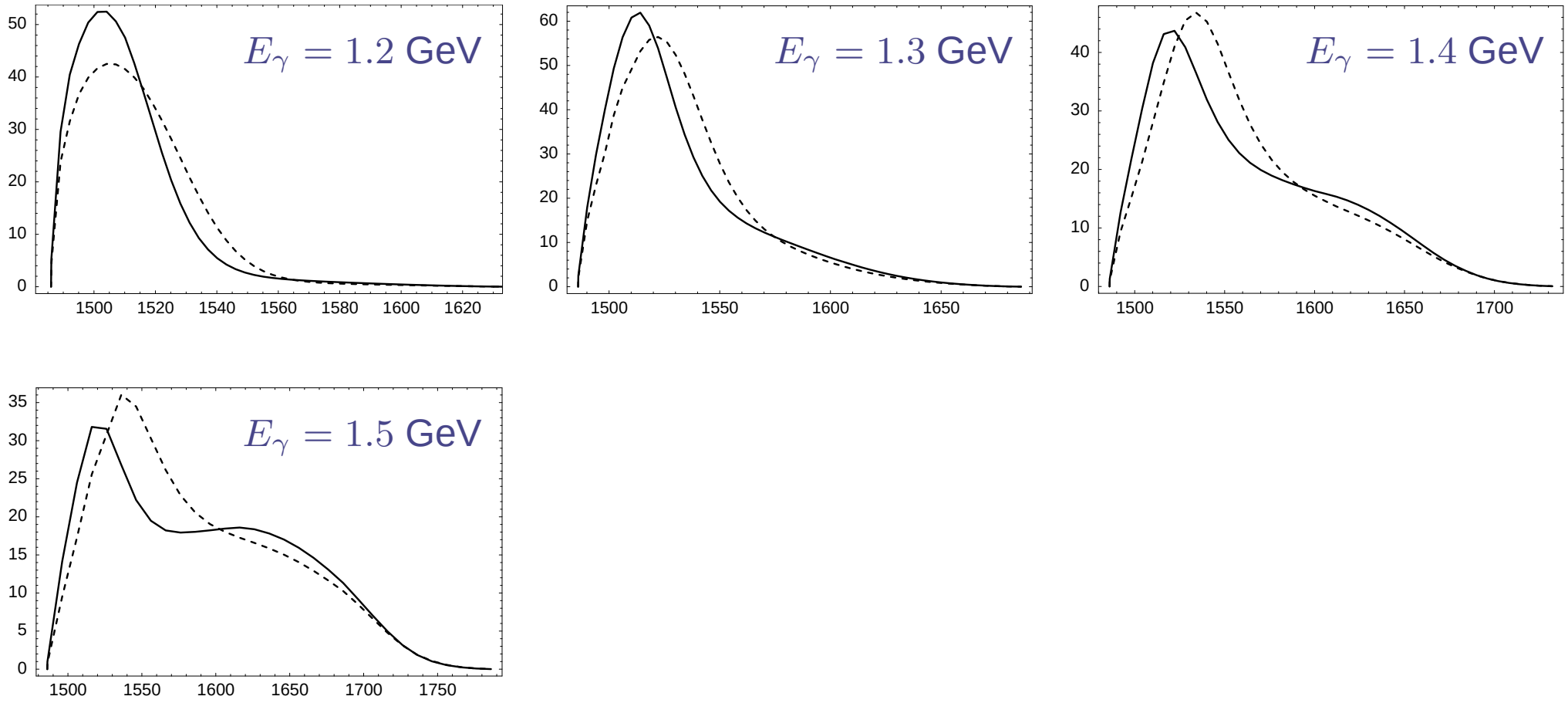

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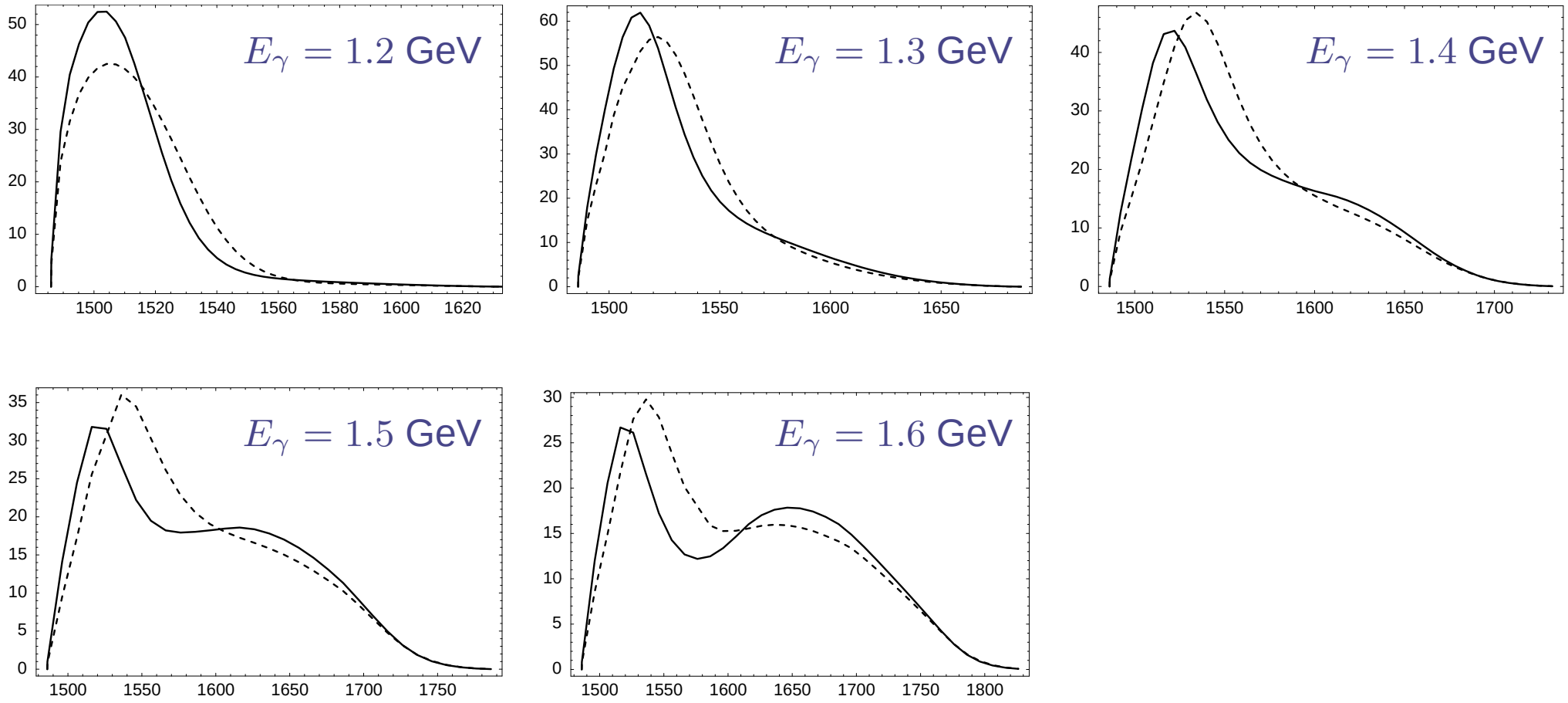

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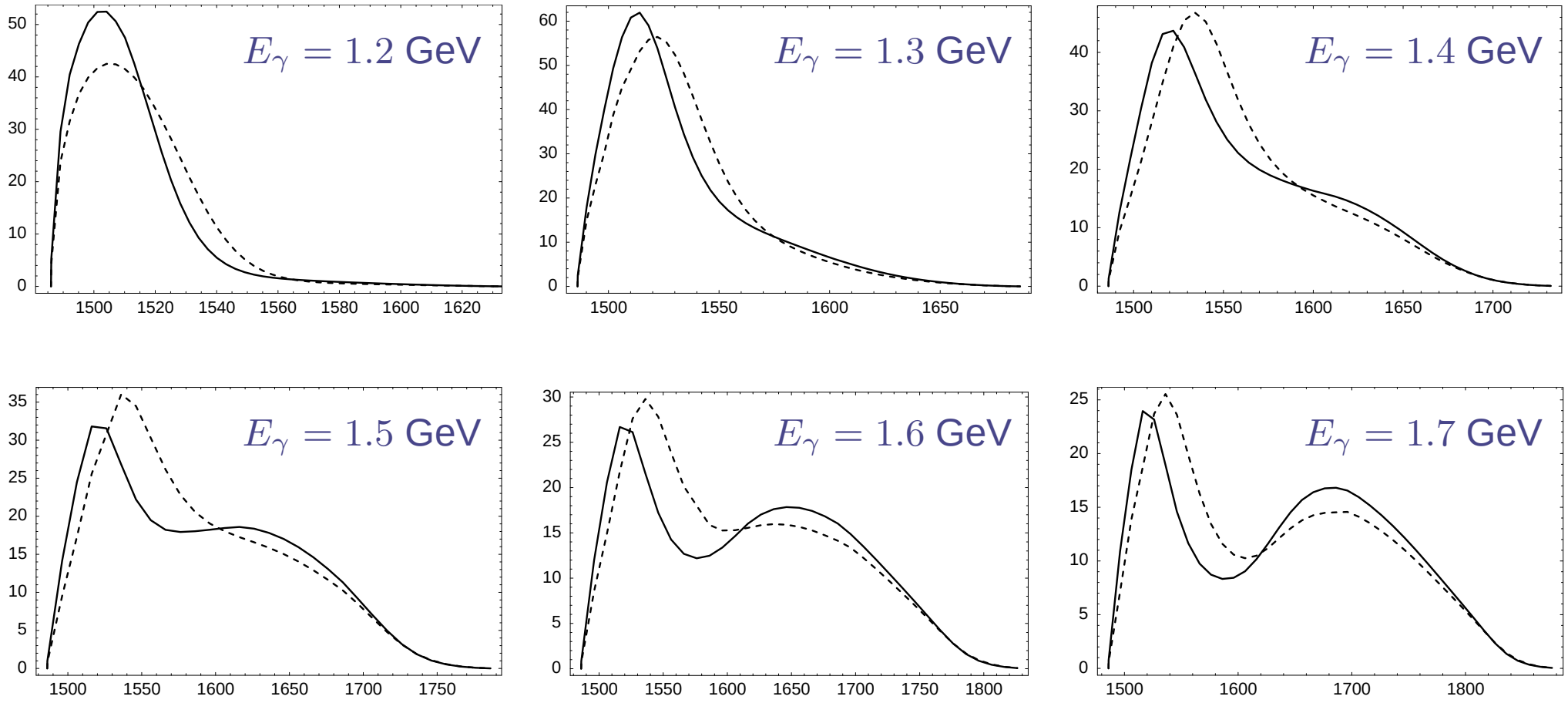

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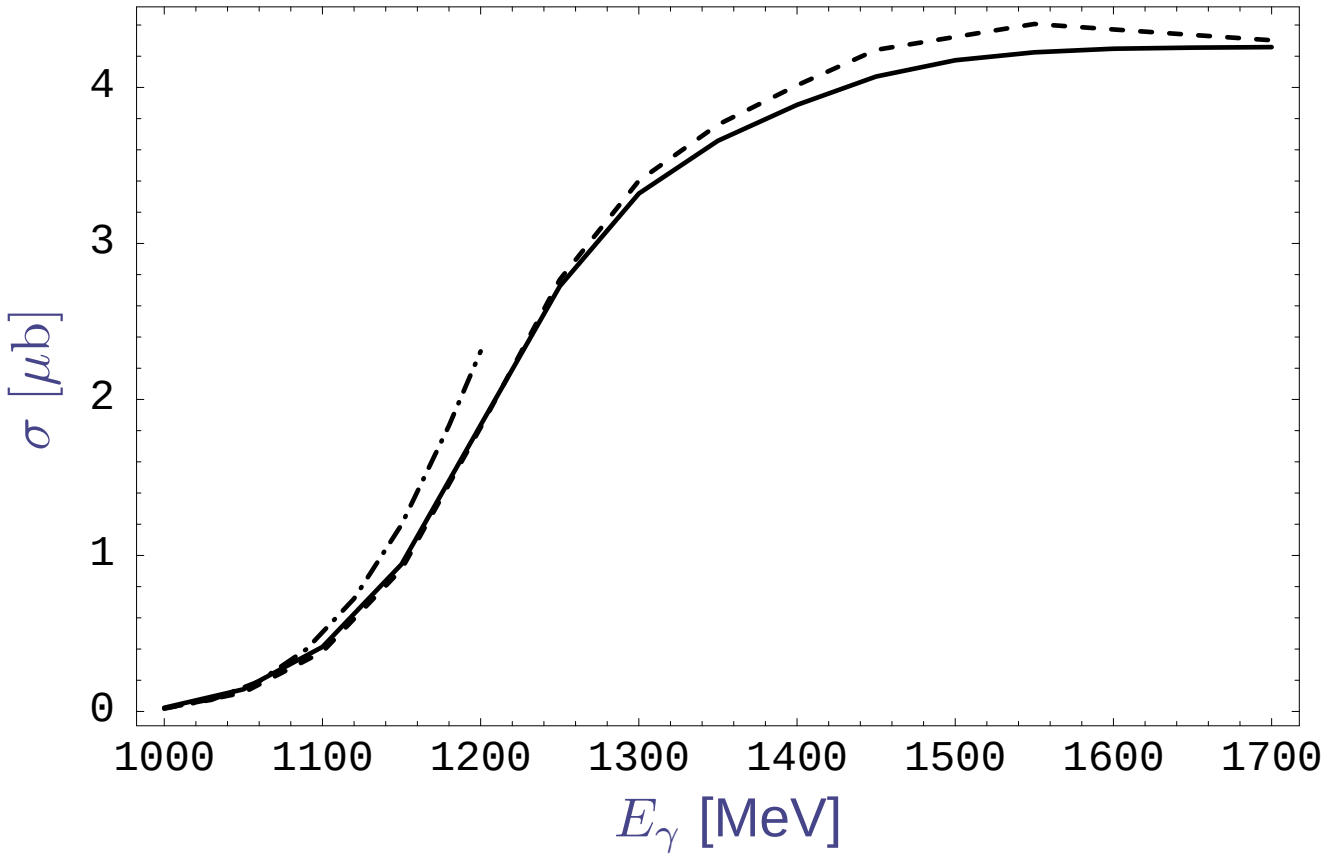
Total cross section for $\gamma p \to \pi^0 \eta p$



Solid line: Full model for the $N^*(1535)$.

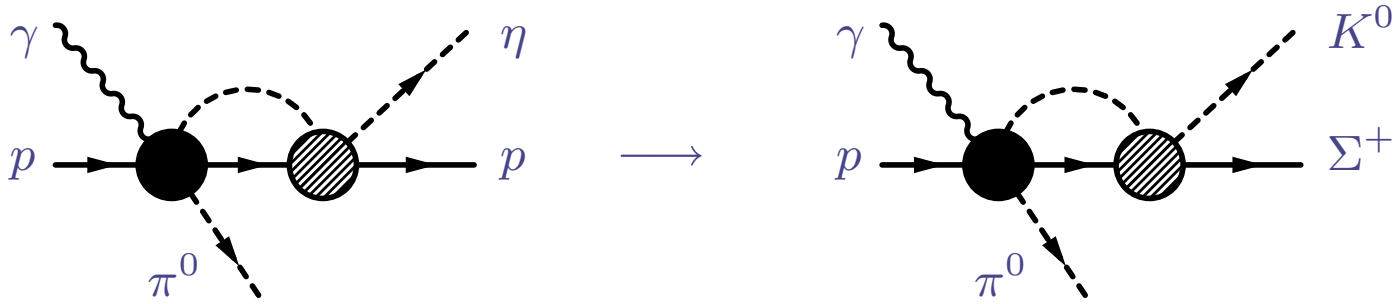
Dashed line: Reduced model.

Dashed dotted line: Phenomenological potential for the $MB \to \eta p$ transition (only available up to $E_\gamma \sim 1.2$ GeV).

In very good agreement with preliminary exp. results (M. Nanova, N^* 2005 conference)

The reaction $\gamma p \to \pi^0 K^0 \Sigma^+$

Coupled Channels: $T^{(i3)} \to T^{(i6)}$:



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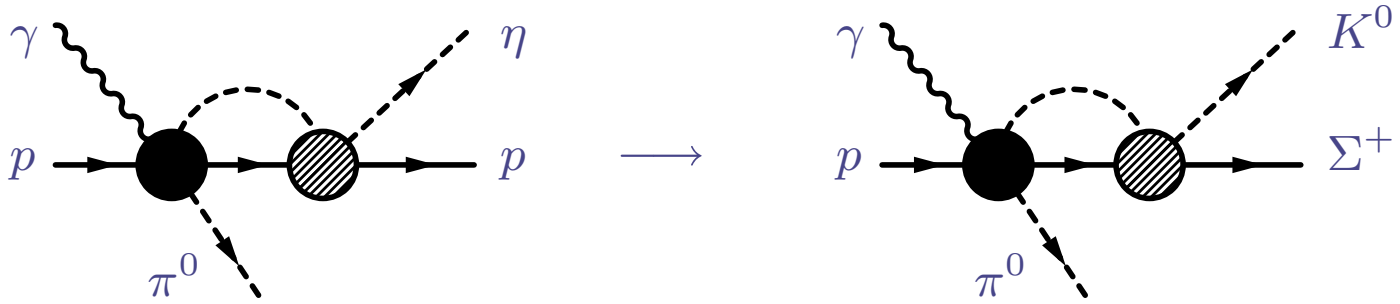
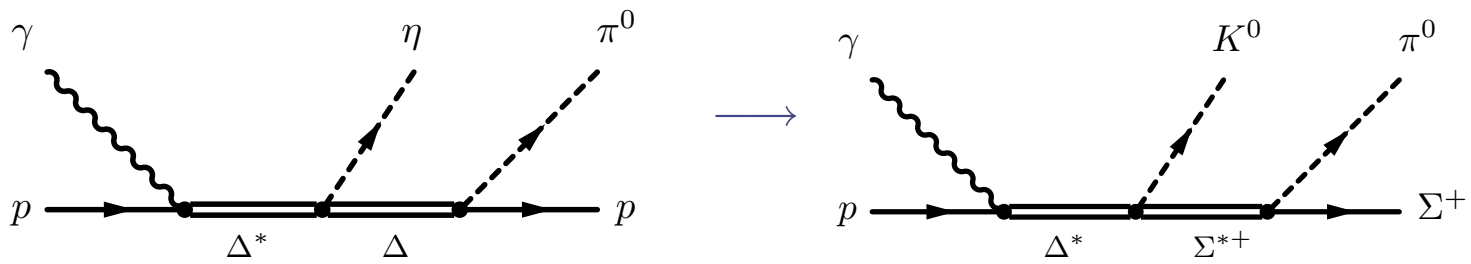
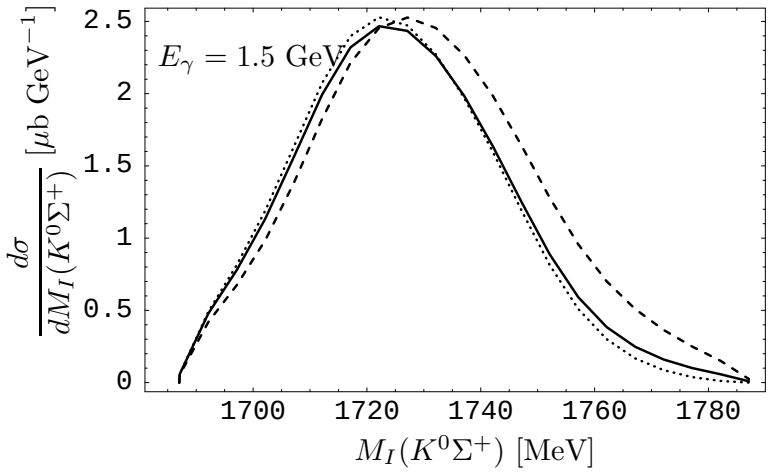


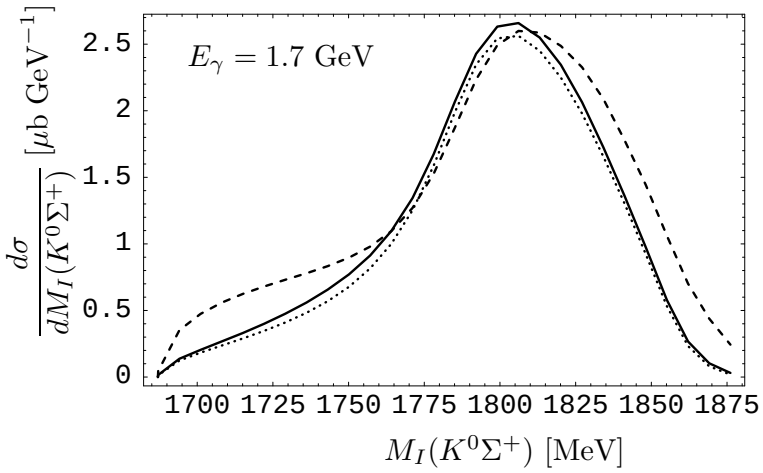
Tree Level:



Results for $\gamma p \to \pi^0 K^0 \Sigma^+$

Invariant mass distribution:





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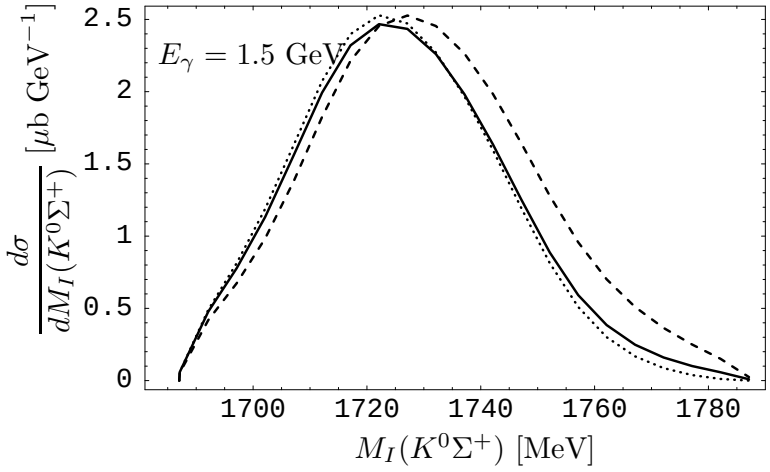
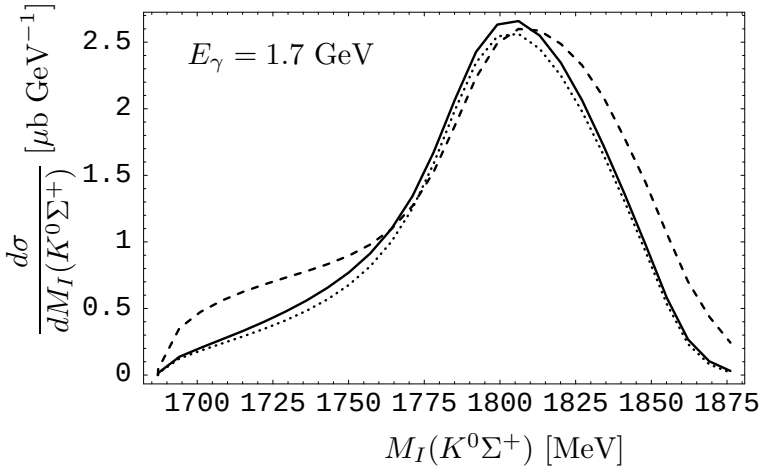
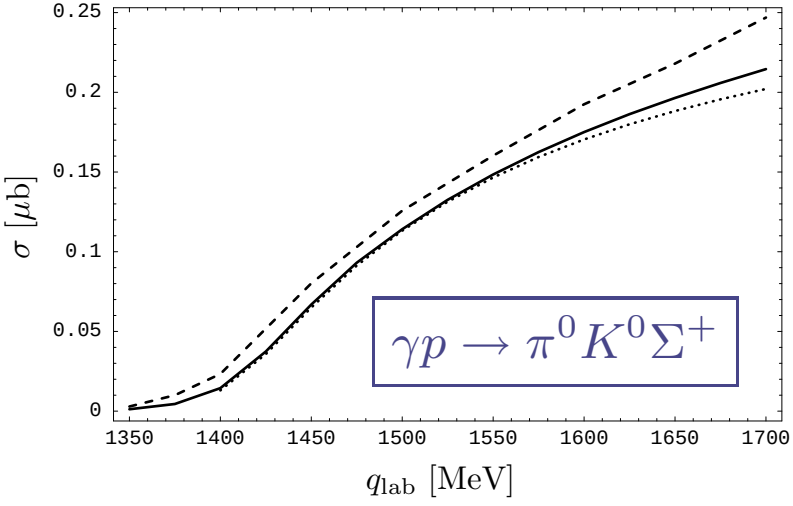




Cross section:



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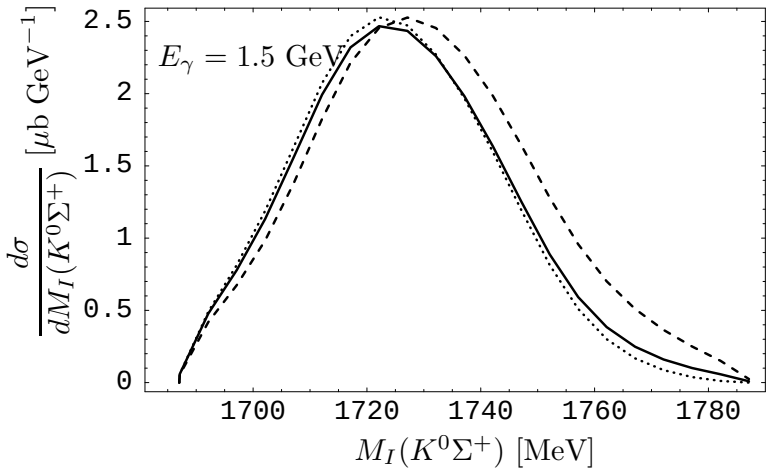

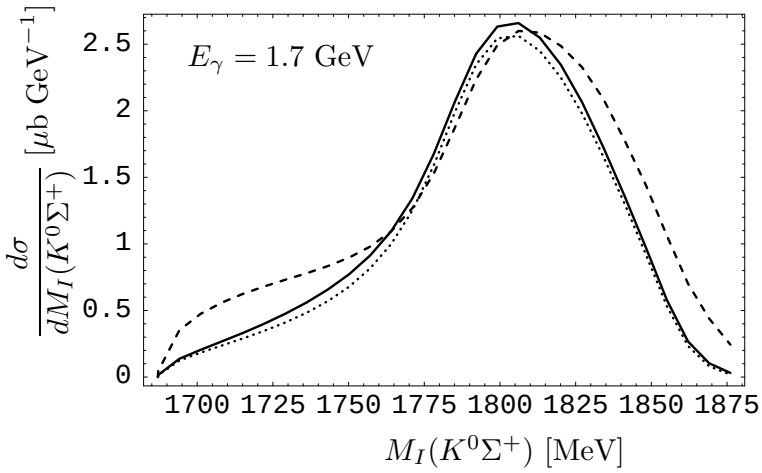

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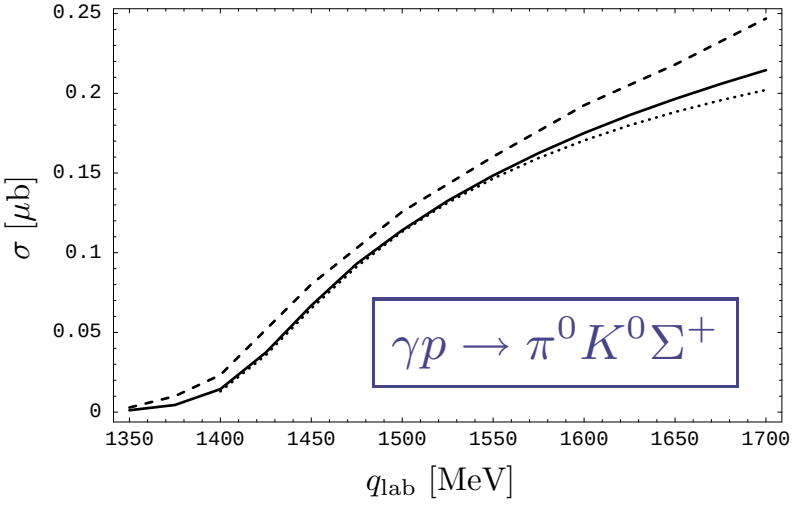

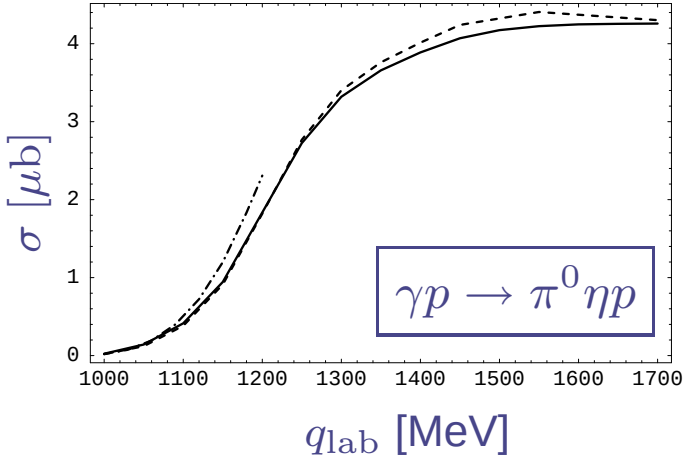

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Additional Slides: Lagrangians

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= \left\langle \bar{B} i\gamma^\mu \nabla_\mu B \right\rangle - M_B \left\langle \bar{B} B \right\rangle \\
&\quad + \frac{1}{2} D \left\langle \bar{B}\gamma^\mu\gamma_5 \left\{ u_\mu, B \right\} \right\rangle + \frac{1}{2} F \left\langle \bar{B}\gamma^\mu\gamma_5 [u_\mu, B] \right\rangle \\
\nabla_\mu B &= \partial_\mu B + [\Gamma_\mu, B], \\
\Gamma_\mu &= \frac{1}{2}(u^+ \partial_\mu u + u \partial_\mu u^+), \\
U &= u^2 = \exp(i\sqrt{2}\Phi/f), \\
u_\mu &= i u^+ \partial_\mu U u^+
\end{aligned}
$$

$$
\mathcal{L}_{\mathrm{BBM}} = \frac{D+F}{2} \left\langle \overline{B}\gamma^\mu\gamma_5 u_\mu B \right\rangle + \frac{D-F}{2} \left\langle \overline{B}\gamma^\mu\gamma_5 B u_\mu \right\rangle
$$

Amplitudes for tree level $\gamma p \to \pi\pi N$

$$
\begin{aligned}
t^{\Delta^*(1700)}_{\gamma p\to\pi^+\pi^0 n} &= -i\,\frac{2}{\sqrt{3}}\,\frac{f^*_{\Delta N\pi}}{m_\pi}\,\vec{S}\cdot\mathbf{p}_\pi\left(\tilde{f}_{\Delta^*\Delta\pi}+\frac{1}{3}\frac{\tilde{g}_{\Delta^*\Delta\pi}}{m_\pi^2}\,q^2\right)G_{\Delta^*}(\sqrt{s})\\
&\quad\left[g_1'\,\frac{\vec{S}^\dagger\cdot\mathbf{k}}{2M}\,(\vec{\sigma}\times\mathbf{k})\vec{\epsilon}-i\vec{S}^\dagger\cdot\vec{\epsilon}\left(g_1'(k^0+\frac{\mathbf{k}^2}{2M})+g_2'\sqrt{s}k^0\right)\right],\\
t^{N^*(1520)}_{\gamma p\to\pi^+\pi^0 n} &= -i\,\frac{\sqrt{2}}{3}\,\frac{f^*_{\Delta N\pi}}{m_\pi}\,\vec{S}\cdot\mathbf{p}_\pi\left(\tilde{f}_{N^{*\prime}\Delta\pi}+\frac{1}{3}\frac{\tilde{g}_{N^{*\prime}\Delta\pi}}{m_\pi^2}\,q^2\right)G_{N^{*\prime}}(\sqrt{s})\\
&\quad\left[g_1\,\frac{\vec{S}^\dagger\cdot\mathbf{k}}{2M}\,(\vec{\sigma}\times\mathbf{k})\vec{\epsilon}-i\vec{S}^\dagger\cdot\vec{\epsilon}\left(g_1(k^0+\frac{\mathbf{k}^2}{2M})+g_2\sqrt{s}k^0\right)\right],\\
t^{\Delta-\mathrm{KR}}_{\gamma p\to\pi^+\pi^0 n} &= \frac{e\sqrt{2}}{9}\left(\frac{f^*_{\Delta N\pi}}{m_\pi}\right)^2(2\mathbf{p}_\pi-i(\vec{\sigma}\times\mathbf{p}_\pi))\cdot\vec{\epsilon}\,F_\pi(q_{on}-k)\left(1-\frac{1}{3}\frac{q^2_{\mathrm{on}}}{q^0_{\mathrm{on}}k^0}\right),\\
t^{\Delta^*(1700)}_{\gamma p\to\pi^0\pi^0 n} &= \frac{1}{2\sqrt{2}}\,t^{\Delta^*(1700)}_{\gamma p\to\pi^+\pi^0 n},\\
t^{N^*(1520)}_{\gamma p\to\pi^0\pi^0 n} &= \sqrt{2}t^{N^*(1520)}_{\gamma p\to\pi^+\pi^0 n},\qquad t^{\Delta-\mathrm{KR}}_{\gamma p\to\pi^0\pi^0 p}=0.
\end{aligned}
$$

$\Delta^*(1700)$ width

$$G_{\Delta^*}(\sqrt{s}) = \frac{1}{\sqrt{s} - M_{\Delta^*} + i\,\frac{\Gamma(\sqrt{s})}{2}}$$

$$\begin{aligned}
\Gamma_{\Delta^*\to N\pi}(\sqrt{s}) &= \Gamma_{\Delta^*\to N\pi}(M_{\Delta^\star})\,\frac{q_{\rm CM}(\sqrt{s})^5}{q_{\rm CM}(M_{\Delta^*})^5}, \\
\Gamma_{\Delta^*\to N\rho[\pi\pi]}(\sqrt{s}) &= \frac{M_N}{6(2\pi)^3}\,\frac{m_{\Delta^*}}{\sqrt{s}}\,g_\rho^2 f_\rho^2 \int d\omega_1\; d\omega_2\; |D_\rho(q_1+q_2)|^2 (\mathbf{q}_1-\mathbf{q}_2)^2\,\Theta(1-|A|), \\
A &= \frac{(\sqrt{s}-\omega_1-\omega_2)^2 - M_N^2 - \mathbf{q}_1^2 - \mathbf{q}_2^2}{2|\mathbf{q}_1||\mathbf{q}_2|}, \\
\Gamma_{\Delta^*\to\Delta\pi[N\pi\pi]} &= \frac{15}{16\pi^2}\int dM_I\;\frac{M_I\;k(M_I)}{4\pi\sqrt{s}}\;\frac{\Gamma_{\Delta\to N\pi}(M_I)\left(|A_s|^2+|A_d|^2\right)}{(M_I-M_\Delta)^2+\left(\frac{\Gamma_{\Delta\to N\pi}(M_I)}{2}\right)^2}\;\Theta(\sqrt{s}-M_I-m_\pi),
\end{aligned}$$

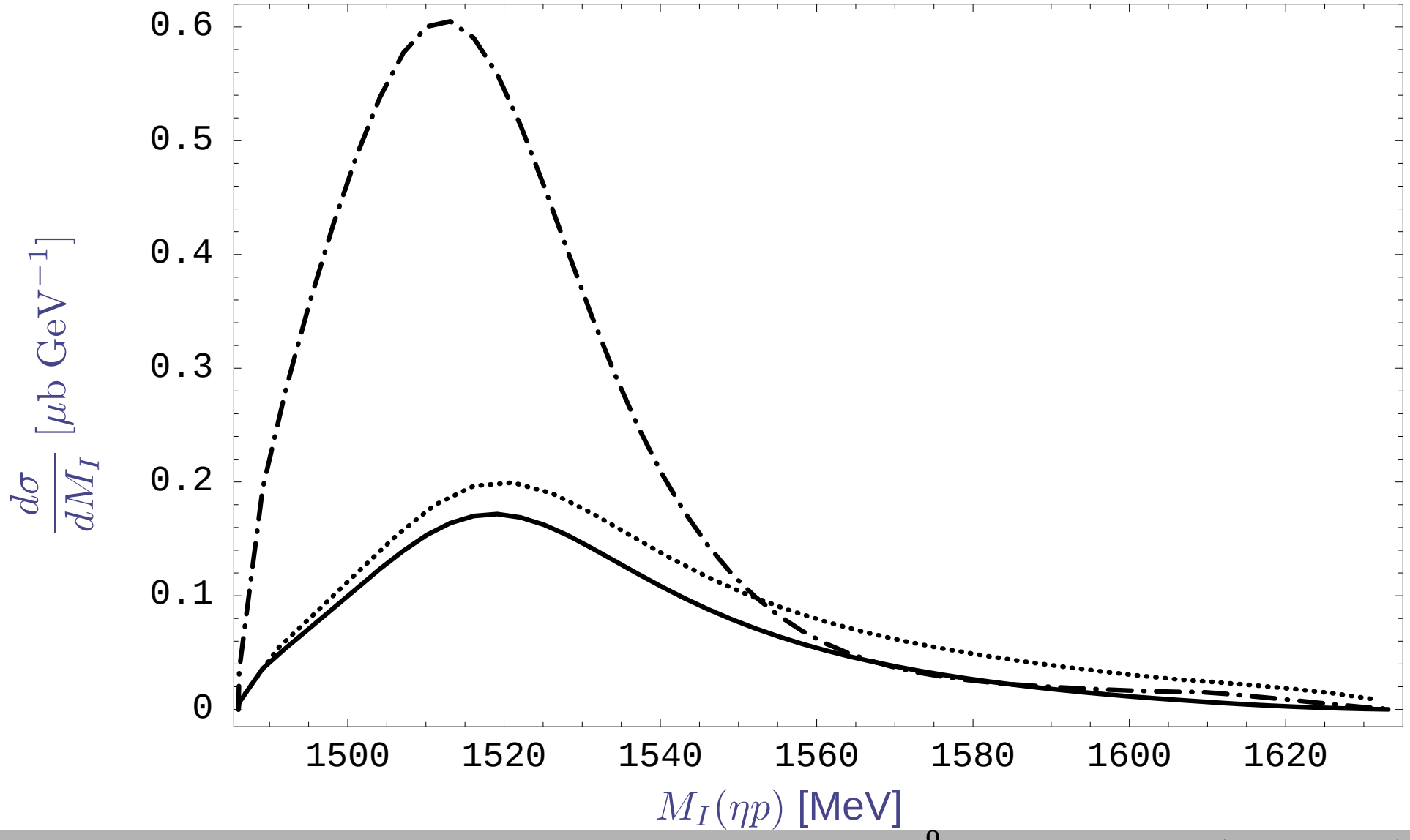
Invariant mass formula

$$\frac{d\sigma}{dM_I(\eta p)} = \frac{1}{4(2\pi)^5}\frac{M_p M_i}{s-M_p^2}\frac{\tilde{p}_\eta p_\pi}{\sqrt{s}}$$

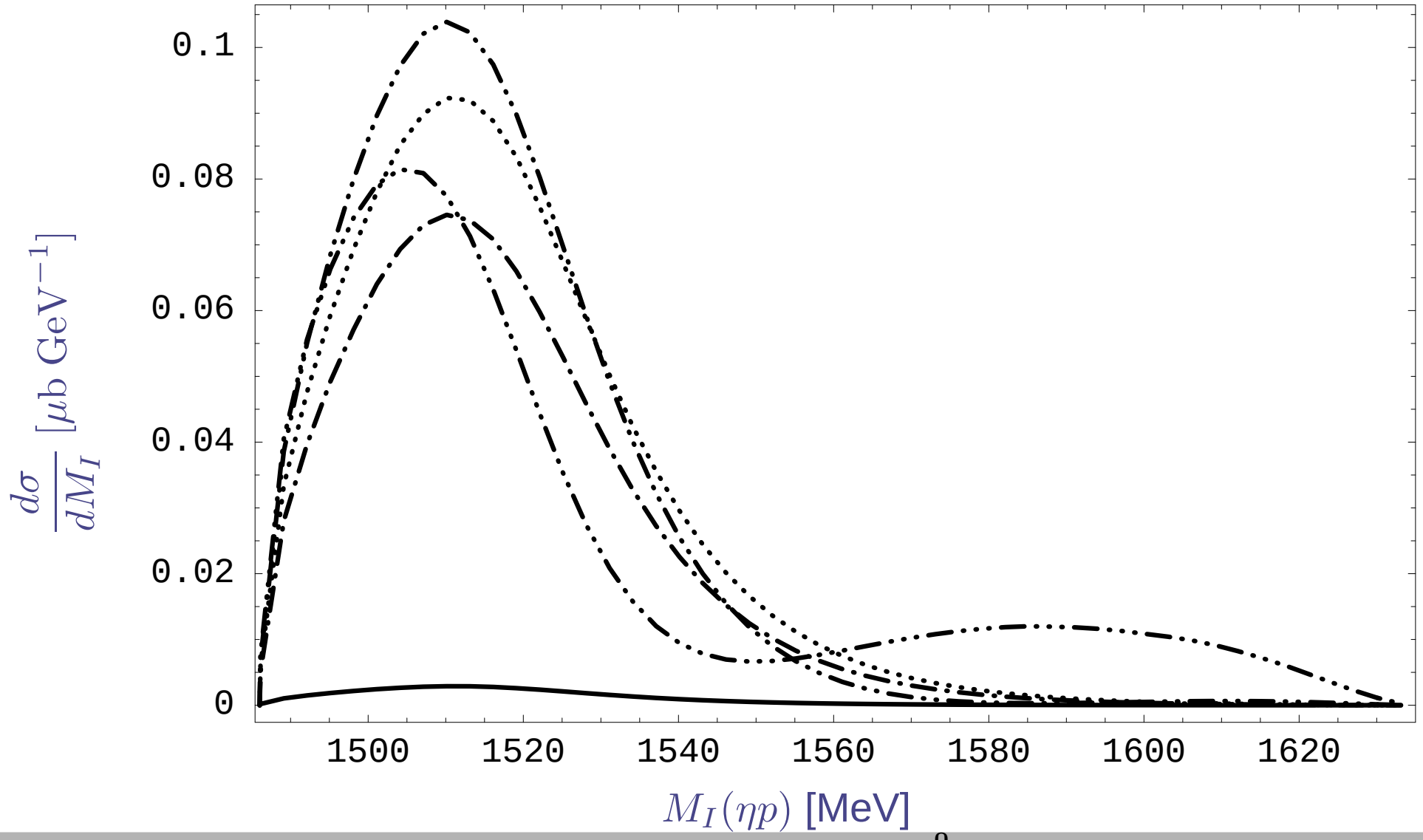
$$\int_0^{2\pi} d\phi_\pi \int_{-1}^{1} d\cos\theta_\pi \int_0^{2\pi} d\tilde{\phi} \int_{-1}^{1} d\cos\tilde{\theta}\, \overline{\sum}\sum |T_{\gamma p\to\pi^0\eta p}|^2,$$

$$\vec{p}_\eta = \left[\left(\frac{\sqrt{s}-\omega_\pi}{M_I}-1\right)\left(-\frac{\vec{\tilde{p}}_\eta \vec{p}_\pi}{\vec{p}_\pi^{\,2}}\right)+\frac{\tilde{p}_\eta^0}{M_I}\right](-\vec{p}_\pi)+\vec{\tilde{p}}_\eta$$

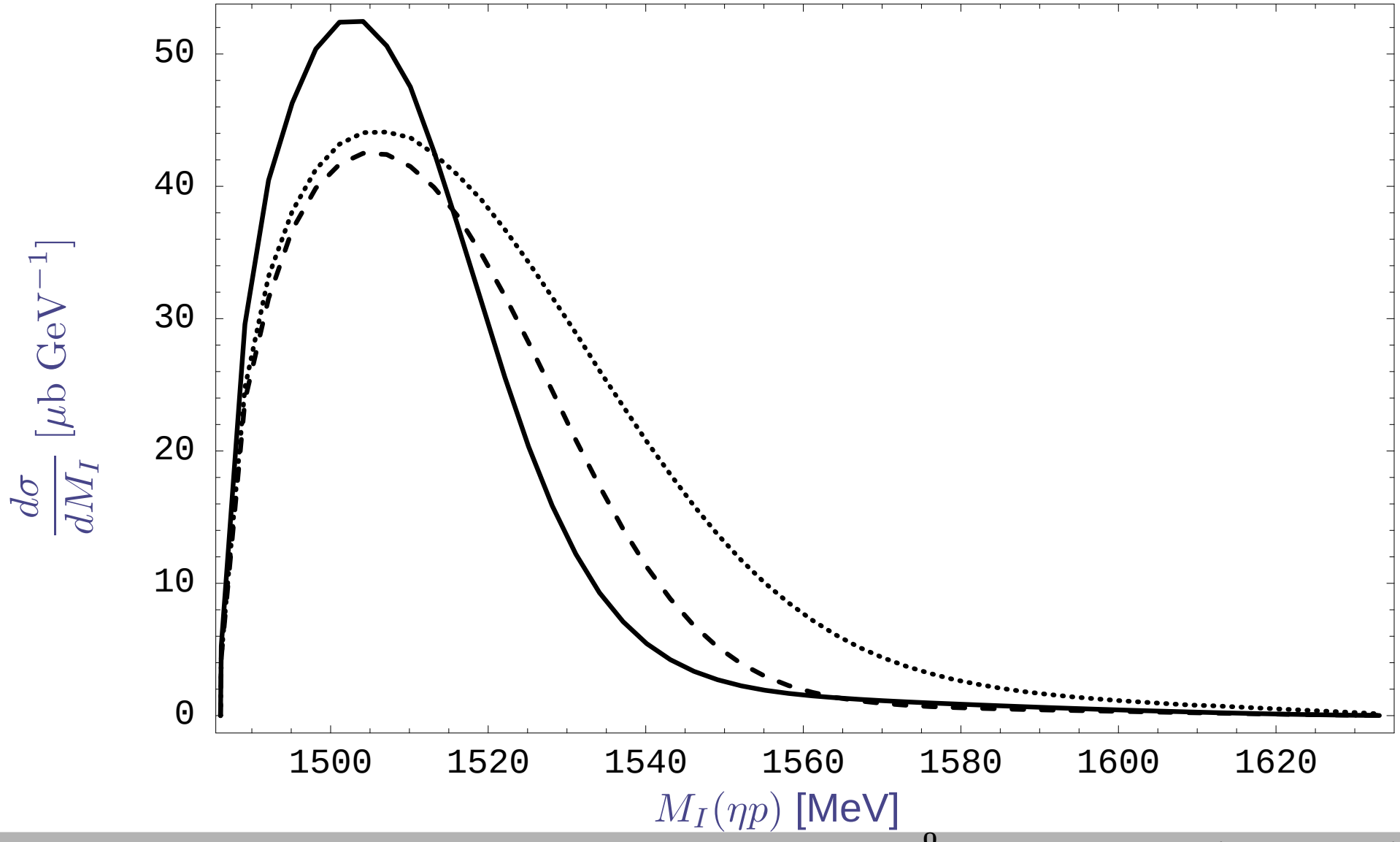
Invariant mass spectra I



Invariant mass spectra II

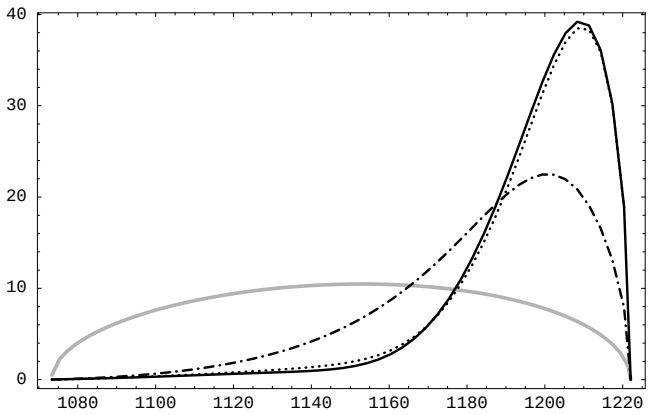


Phenomenological $\pi N \to \eta N$ transition



The $\pi^0 p$ invariant mass in $\gamma p \to \pi^0 \eta p$

$E_\gamma = 1.2$ GeV



Invariant mass spectrum $\frac{d\sigma}{dM_I(\pi^0 p)}$ $[\mu\mathrm{b}\ \mathrm{GeV}^{-1}]$ as a function of $M_I(\pi^0 p)$ [MeV].

Solid lines: Full model for the $N^*(1535)$.

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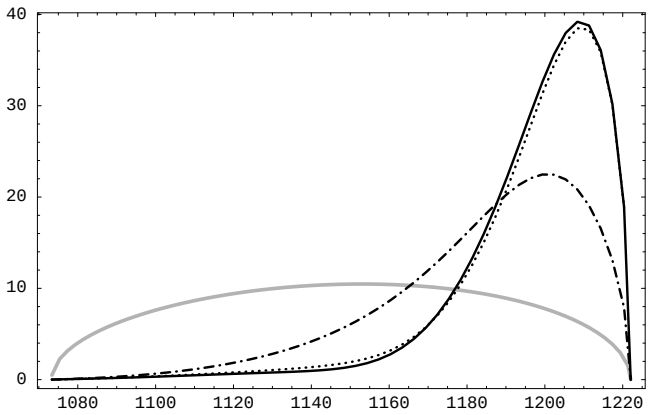

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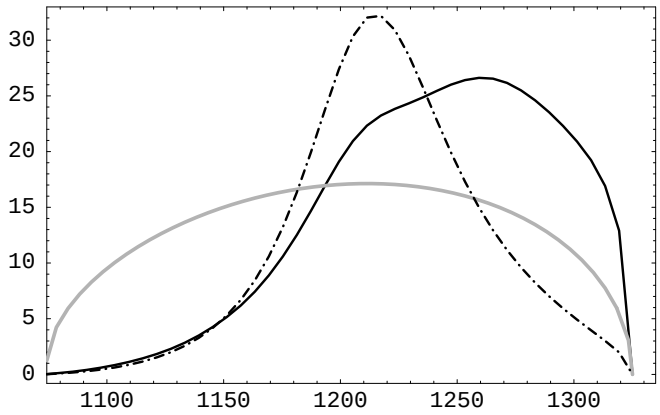

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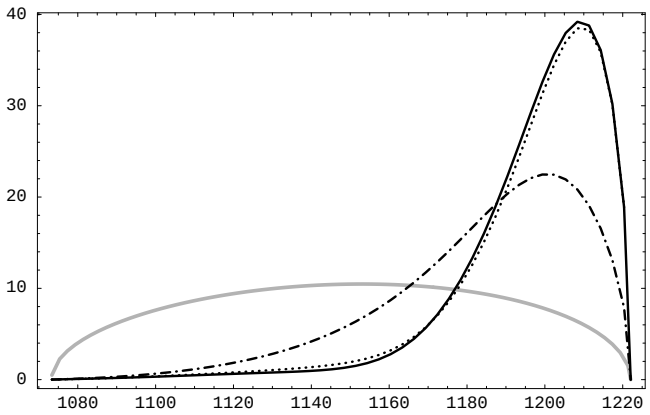

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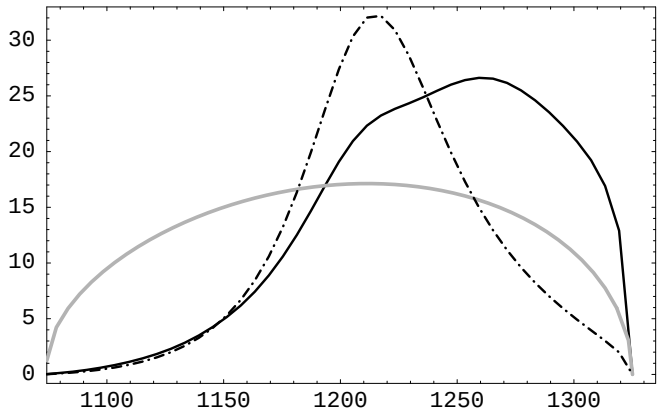
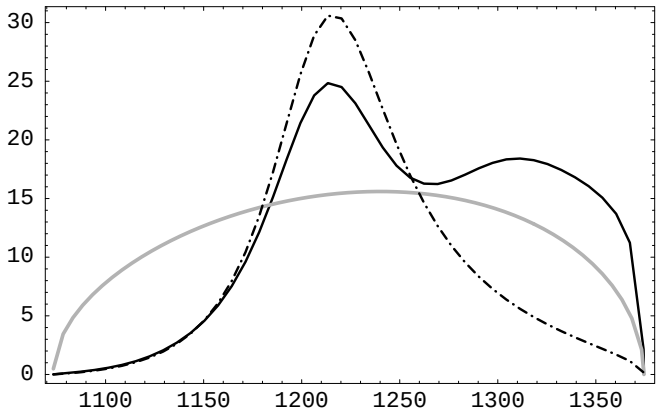


$E_\gamma = 1.5$ GeV



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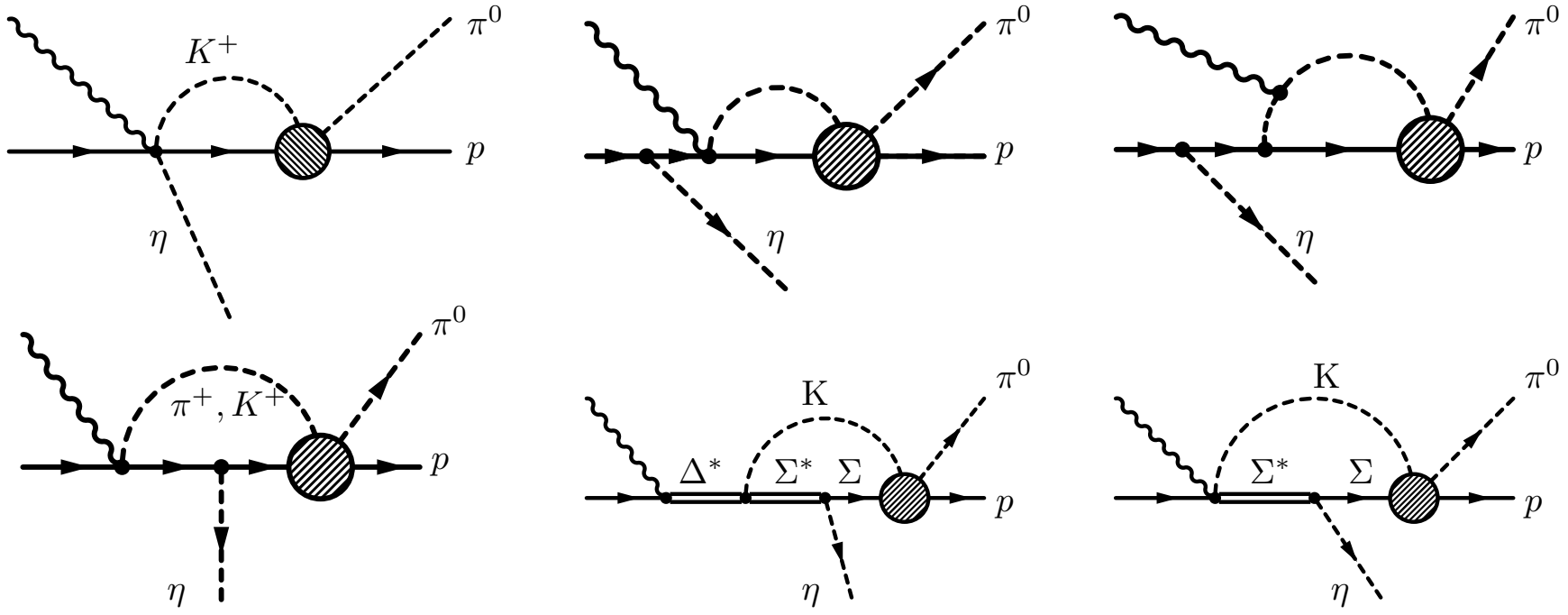
Suppressed diagrams