Fragmentation, NRQCD and Factorization in Heavy Quarkonium Production

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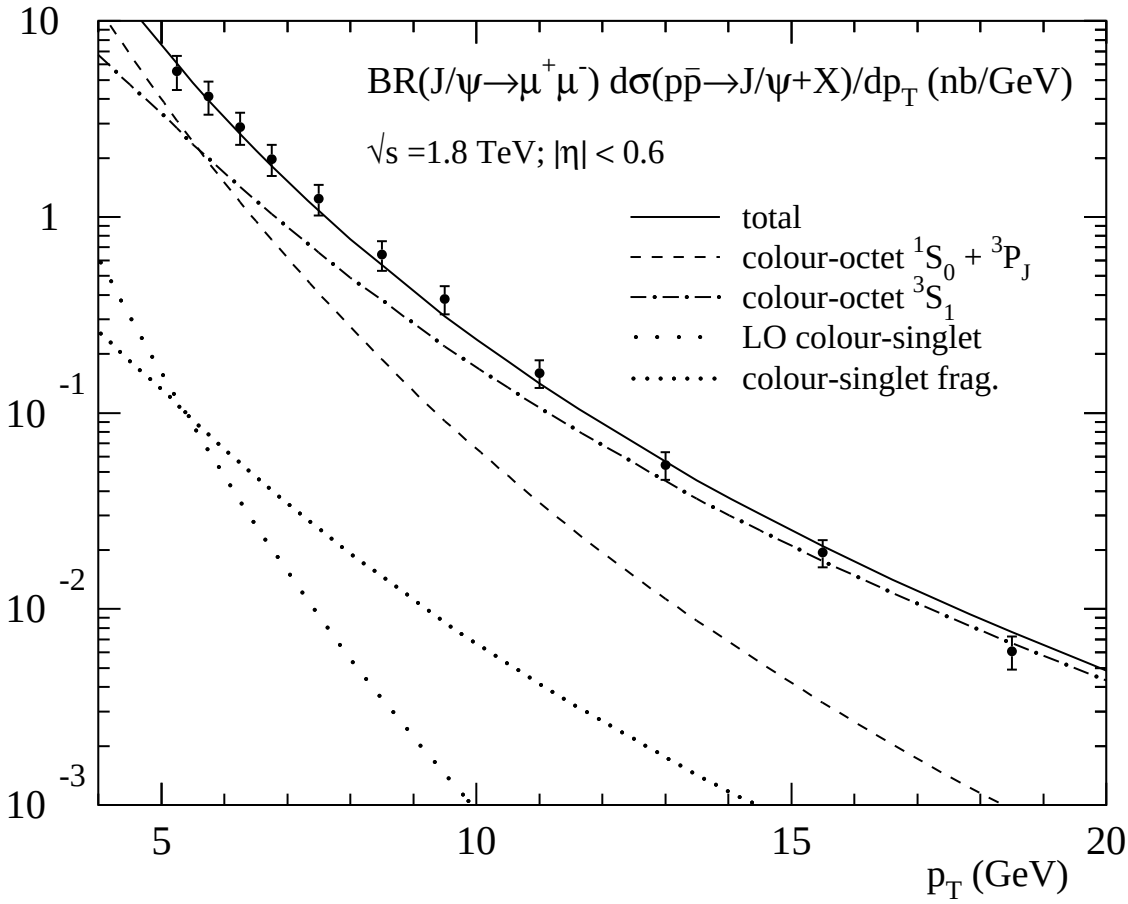
Heavy Quarkonium Production Mechanism at Fixed Target And Collider Experiments

Non-Relativistic QCD (NRQCD)

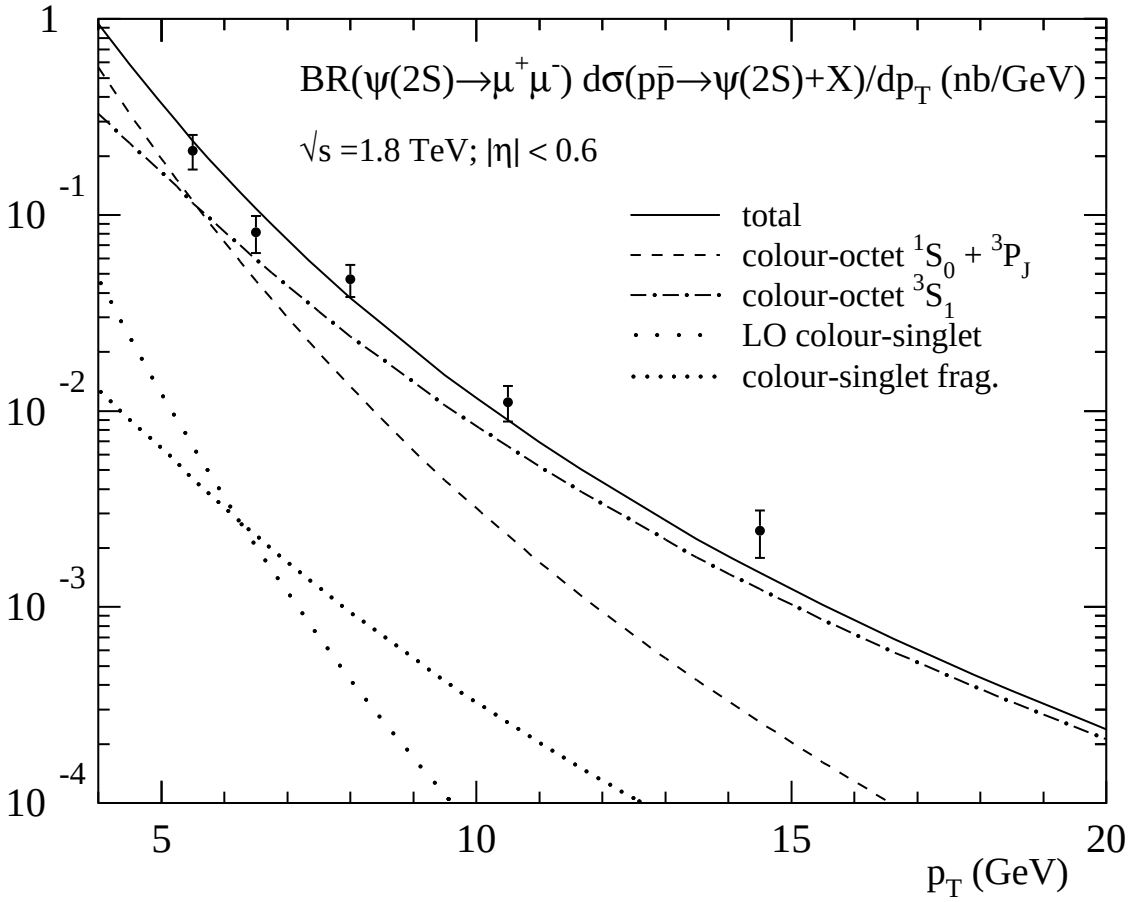
Color Singlet Mechanism:

Color Octet Mechanism:

Failure Of Color Singlet Mechanism at Tevatron



J/Ψ Production at Tevatron (Failure of Color Singlet Mechanism)



$\Psi(2S)$ Production at Tevatron (Failure of Color Singlet Mechanism)

Color Octet Mechanism
(Bodwin, Braaten, Lepage (1994))

Two Parameters:

Coupling Constant: $\alpha_s(M)$**:**

Relative Velocity in $Q\bar{Q}$ **Bound State:** v**:**

$\alpha_s(2M_c) \sim 0.24$, $\frac{v^2}{c^2} \sim 0.23$, **For** $C\bar{C}$ **System**

$\alpha_s(2M_b) \sim 0.18$, $\frac{v^2}{c^2} \sim 0.08$, **For** $B\bar{B}$ **System**

1) M**: Quark Mass: Scale For Annihillation Decays.**

2) Mv**: Momentum: Length Scale For the Size of the Quarkonium States.**

3) Mv^2**: Kinetic Energy: Scale of Splitting Between Different Excitation Sates (Both Radial and Angular)**

NRQCD LAGRANGIAN DENSITY

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{light} + \mathcal{L}_{heavy} + \mathcal{L}_{correction}$$

$$\mathcal{L}_{light} = -\tfrac{1}{2} F^{a\mu\nu} F^{a}_{\mu\nu} + \Sigma\, \bar{q}\gamma_\mu D^\mu[A]q$$

In Terms of Two component Dirac Spinors:

$$\Psi_{heavy} = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

$$\mathcal{L}_{heavy} = \psi^\dagger(iD_t + \tfrac{D^2}{2M})\psi + \chi^\dagger(iD_t + \tfrac{D^2}{2M})\chi$$

$$\mathcal{L}_{correction} = \tfrac{1}{8M^3}[\psi^\dagger D^4\psi - \chi^\dagger D^4\chi]$$
$$+ \tfrac{g}{8M^2}[\psi^\dagger(D\cdot E - E\cdot D)\psi + \chi^\dagger(D\cdot E - E\cdot D)\chi]$$
$$+ \tfrac{ig}{8M^2}[\psi^\dagger\sigma\cdot(D\times E - E\times D)\psi + \chi^\dagger\sigma\cdot(D\times E - E\times D)\chi]$$
$$+ \tfrac{g}{2M}[\psi^\dagger\sigma\cdot B\psi - \chi^\dagger\sigma\cdot B\chi]$$

Heavy Quarkonium Production Amplitude is:

$$|\psi_Q >= O(1)|Q\bar{Q}[^3S_1^{(1)}] > +O(v)|Q\bar{Q}[^3P_J^{(8)}]g > + O(v^2)|Q\bar{Q}[^3S_1^{(1,8)}]gg > +O(v^2)|Q\bar{Q}[^1S_0^{(8)}]g > + O(v^2)|Q\bar{Q}[^3D_J^{(1,8)}]gg > +.....$$

and the wave functions of P-wave ortho-quarkonium state $|\chi_{QJ} >$:

$$|\chi_{QJ} >= O(1)|Q\bar{Q}[^3P_J^{(1)}] > +O(v)|Q\bar{Q}[^3S_1^{(8)}]g > +....$$

Fragmentation Function For a Parton to Heavy Quarkonium is:

$$d\sigma_{A+B\to H+X}(p_T) = \Sigma_n \; d\hat{\sigma}_{A+B\to c\bar{c}[n]+X}(p_T) \, \langle \mathcal{O}_n^H \rangle$$

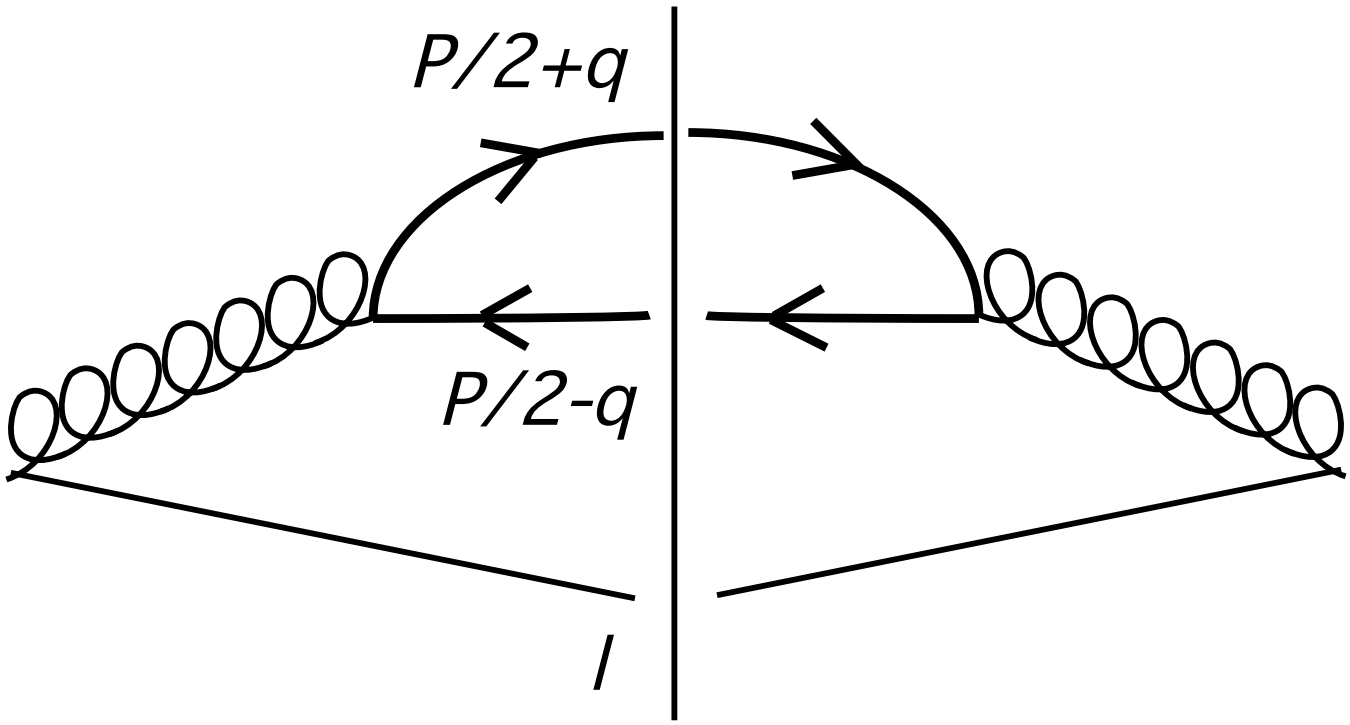
The Fragmentation Function is:

$$D_{H/i}(z, m_c, \mu) = \Sigma_n \; d_{i\to c\bar{c}[n]}(z, \mu, m_c) \, \langle \mathcal{O}_n^H \rangle$$

The NRQCD Non-Perturbative Matrix Element is:

$$\mathcal{O}_n^H(0) = \Sigma_N \; \chi^\dagger(0)\kappa_n\psi(0) \; |N, H\rangle \; \langle N, H| \; \psi^\dagger(0)\kappa_n'\chi(0)$$
$$= \chi^\dagger(0)\kappa_n\psi(0) \; \left(a_H^\dagger a_H\right) \; \psi^\dagger(0)\kappa_n'\chi(0)$$

The Coefficient Function $d_{i\to c\bar{c}[n]}(z, \mu, m_c)$ Can be Calculated Perturbatively

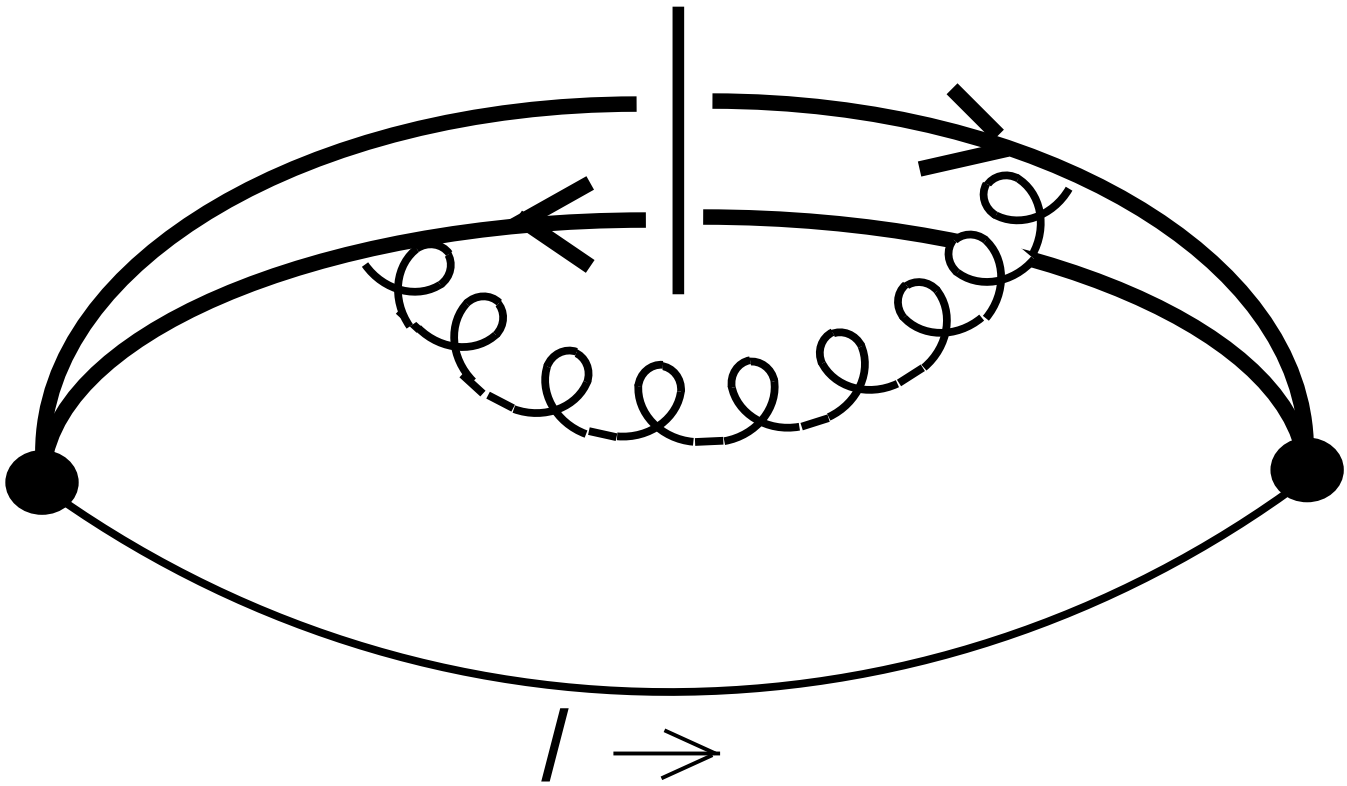


(a)

Fragmentation Function at Leading Order (LO)

Proof Of Factorization in Heavy Quarkonium Production in Hadronic Collisions

- **Soft gluons and infrared divergences in heavy quarkonium production at high p_T**

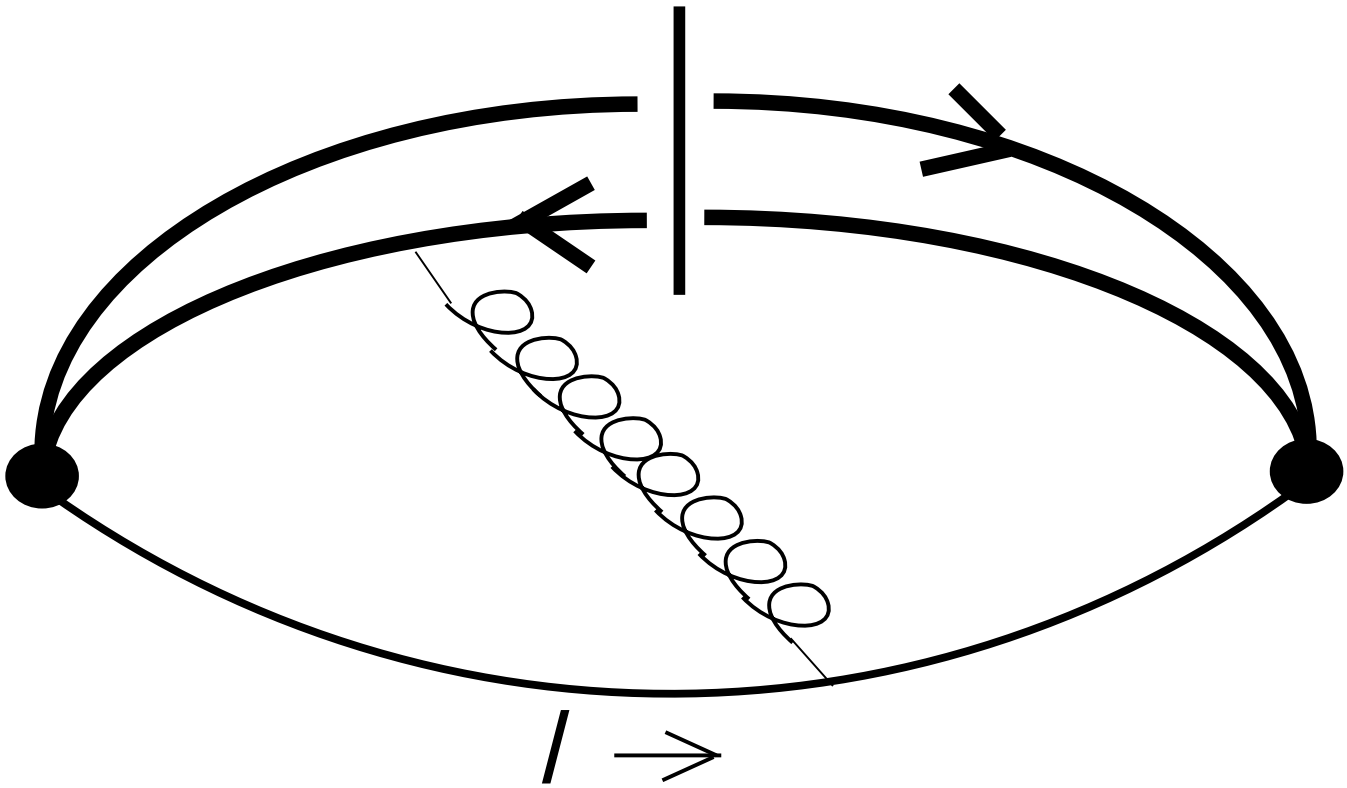


(b)

Fragmentation Function at Next to Leading Order (NLO) and order q^2

$$\Sigma^{NLO}(P,q) \;=\; 16\, g^2 \mu^{2\varepsilon} \int \frac{d^D k}{(2\pi)^{D-1}}\; \delta(k^2)[q_\nu (P\cdot k) - (q\cdot k) P_\nu] \times \; [q^\nu (P\cdot k) - (q\cdot k) P^\nu] \frac{1}{[(P\cdot k)^2]^2}$$
$$= \frac{16}{3}\, \frac{\alpha_s}{\pi}\, \frac{\vec{q}^{\,2}}{P^2}\, \frac{1}{-\varepsilon} + \ldots$$

Infrared divergences found, but can be absorbed into conventional NRQCD matrix elements; topologically factorized

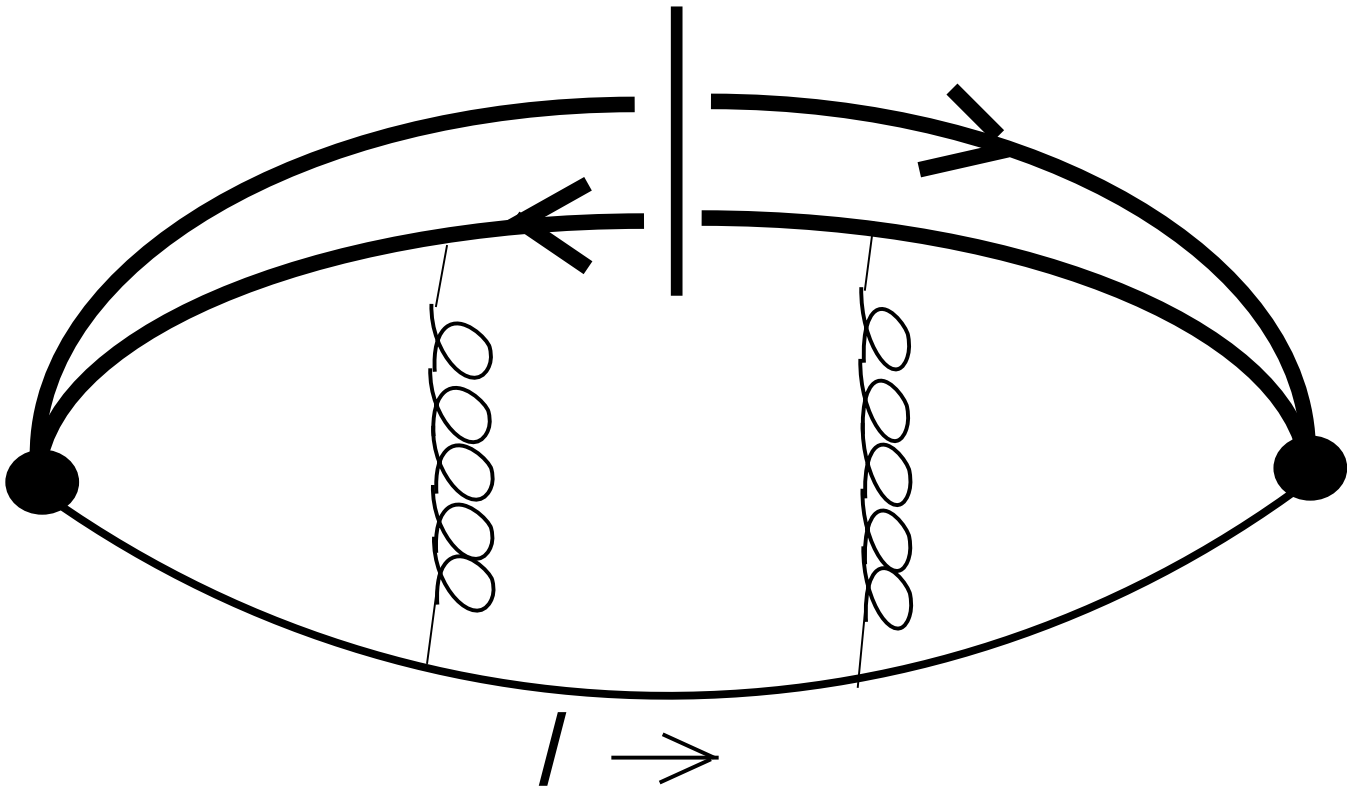


(c)

From this diagram the the divergences exactly cancel from the virtual and real contributions

Hence at NLO, the infrared divergences cancel except when topologically factorized, consistent with conventional NRQCD matrix elements

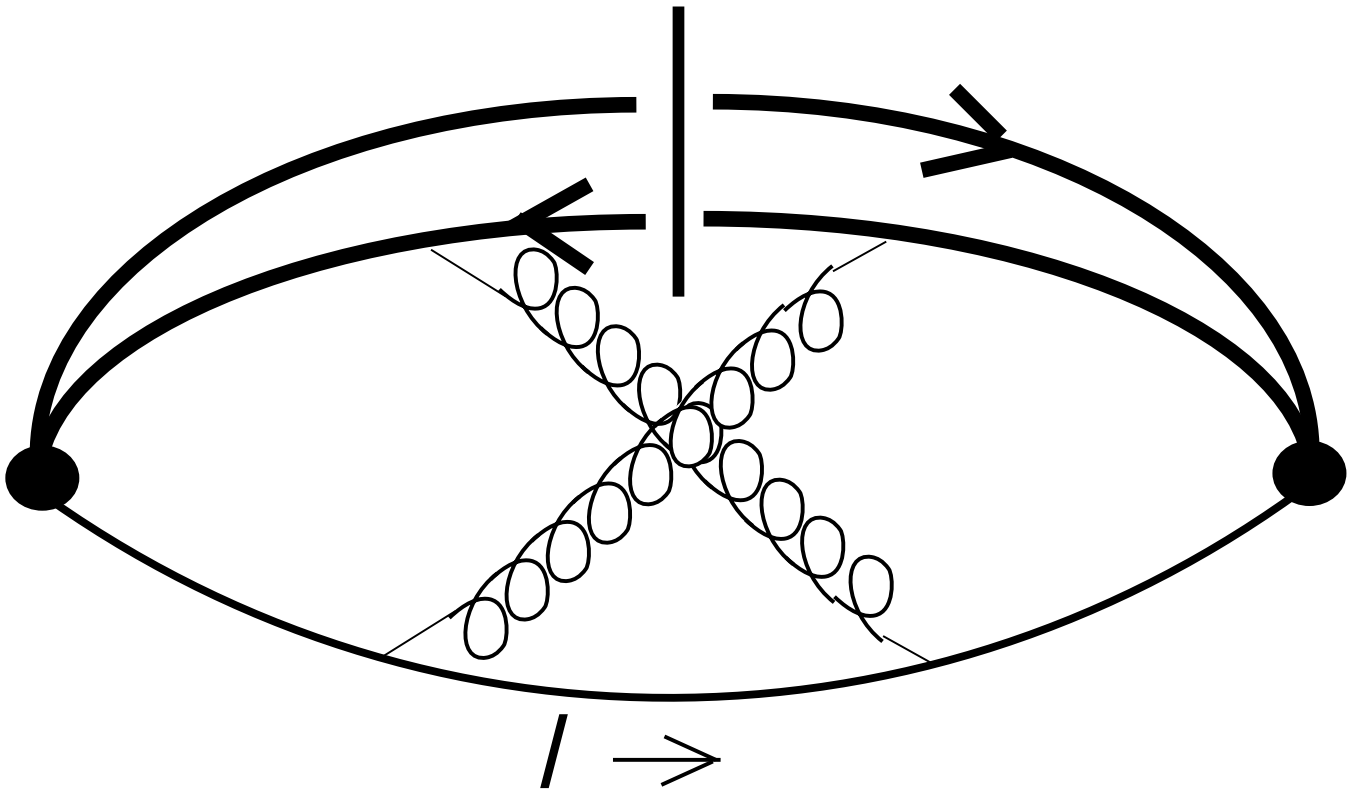
$\mathcal{O}_n^H(0) = \chi^\dagger(0)\kappa_n\psi(0)\left(a_H^\dagger a_H\right)\psi^\dagger(0)\kappa_n'\chi(0)$



(a)

NNLO Diagrams

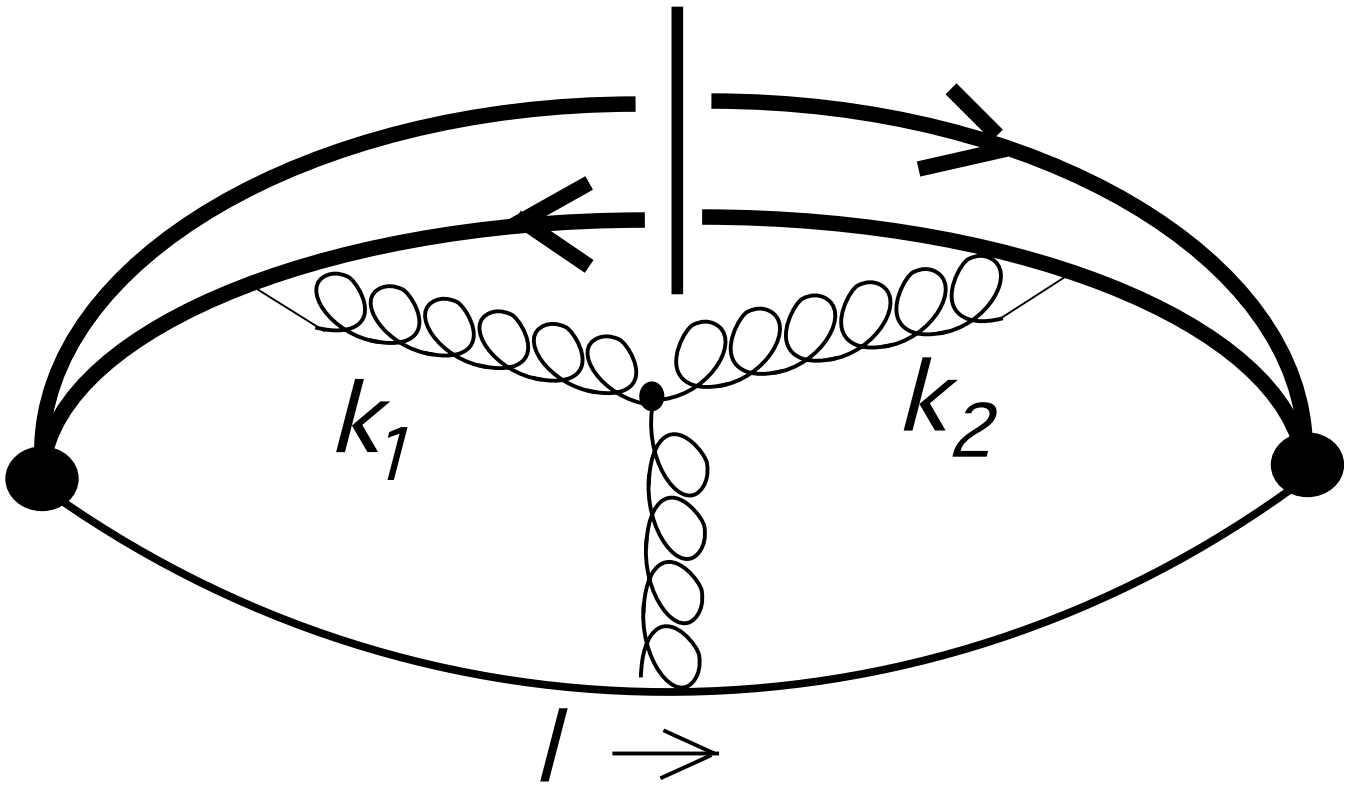
The Real part of these diagrams does not contain infrared divergences



(b)

NNLO Diagrams

The Real Part of these diagrams does not contain infrared divergences



(c)

NNLO Diagrams with Three Gluon Vertex:

Infrared Divergence Is Present in These Diagrams

$$
\begin{aligned}
&\Sigma^{(2c)}(P,q,l) \;=\\
&-16i\, g^4 \mu^{4\varepsilon} \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D}\; 2\pi\; \delta(k_1^2)\; l^\lambda\, V_{\nu\mu\lambda}[k_1,k_2]\\
&\times\; [q^\mu (P\cdot k_1) - (q\cdot k_1) P^\mu]\;\; [q^\nu (P\cdot k_1) - (q\cdot k_2) P^\nu]\\
&\times \frac{1}{[P\cdot k_1 + i\epsilon]^2\; [P\cdot k_2 - i\epsilon]^2}\\
&\times \frac{1}{[k_2^2 - i\epsilon]\; [(k_2-k_1)^2 - i\epsilon]\; [l\cdot(k_1-k_2) - i\epsilon]}
\end{aligned}
$$

$$
\Sigma^{(2)}(P,q,l) \;=\; \alpha_s^2\, \frac{4}{3\varepsilon}\, [\frac{(P\cdot q)^2}{P^4} \;-\; \frac{q^2}{P^2}]
$$

In the rest frame of heavy-quarkonium ($\vec{P}=0$), this becomes simply

$$
\Sigma(P,q,l) \;=\; \alpha_s^2\, \frac{4}{3\varepsilon}\, \frac{\vec{q}^{\,2}}{4m_c^2} \;=\; \alpha_s^2\, \frac{1}{3\varepsilon}\, \frac{\vec{v}^{\,2}}{4}
$$

- **Infrared Divergences Found at NNLO**

- **Breakdown of the simplest topological factorization**
- **Conclude: we need the Wilson lines**

  Redefinition NRQCD Matrix Elements
- **Resolution: as for fragmentation, supplement fields by Wilson lines:**

$$\Phi_l[x, A] = \exp\left[-ig \int_0^\infty d\lambda\, l \cdot A(x + \lambda l)\right]$$

- **Our new, gauge-invariant NRQCD Non-Perturbative Matrix Element:**

$$\mathcal{O}_n^H(0) \to \chi^\dagger \mathcal{K}_{n,c} \psi(0)\, \Phi_l^\dagger[0, A]_{cb} \left(a_H^\dagger a_H\right)\, \Phi_l[0, A]_{ba}\, \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$$

- **Conventional NRQCD Non-Perturbative Matrix Element was:**

$$\mathcal{O}_n^H(0) \to \chi^\dagger \mathcal{K}_{n,a} \psi(0)\, \left(a_H^\dagger a_H\right)\, \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$$

CONCLUSIONS

We have studied Factorization in Heavy Quarkonium Production Using NRQCD

We Found Infrared Divergences at NNLO

Resolution: as for fragmentation, supplement fields by Wilson lines:

Redefinition NRQCD Matrix Elements

$$\mathcal{O}_n^H(0) \equiv \chi^\dagger \mathcal{K}_{n,a} \psi(0) \left(a_H^\dagger a_H\right) \chi^\dagger \mathcal{K}'_{n,a} \psi(0) \rightarrow$$
$$\chi^\dagger \mathcal{K}_{n,c} \psi(0)\, \Phi_l^\dagger[0,A]_{cb} \left(a_H^\dagger a_H\right) \Phi_l[0,A]_{ba}\, \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$$