PROBING THE PHASES OF QCD MATTER

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WHAT IS QCD?

A theory of quarks and gluons....

WHAT DOES QCD DESCRIBE?

Colorless, heavy, hadrons...

Hadrons are the (rather complicated) quasi-particles of the QCD vacuum.

The vacuum, whose excitations are the hadrons, is therefore quite a nontrivial [confinement; chiral symmetry breaking; strong coupling; ...] phase of the theory.

BUT: QCD is asymptotically free ....

DO OTHER (SIMPLER?) PHASES EXIST?

Do other phases exist whose quasiparticles look more like the quarks and gluons of the QCD Lagrangian? And look more like phases familiar from QED?

Asymptotic freedom: quarks and gluons weakly interacting

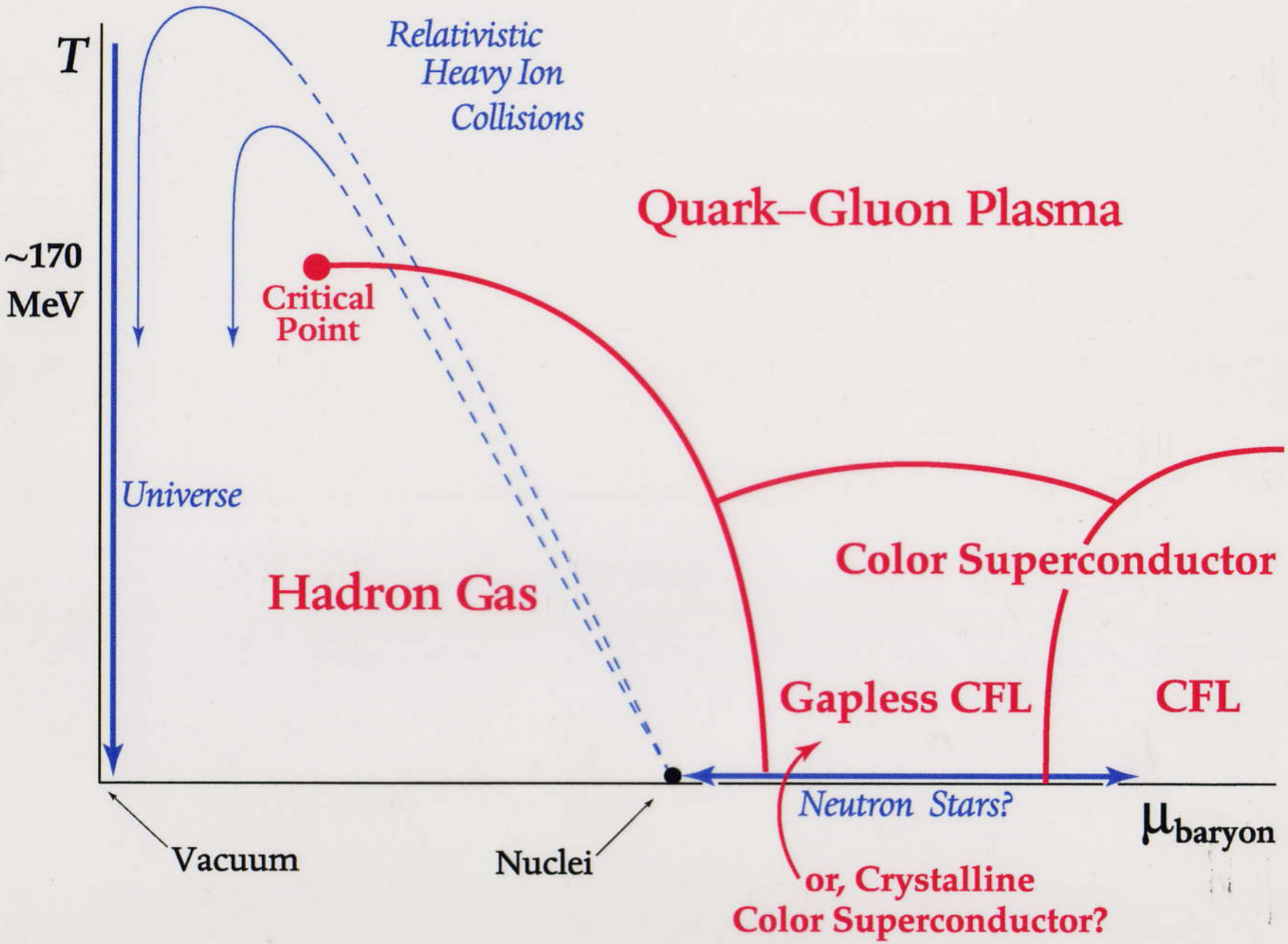
i) when close together

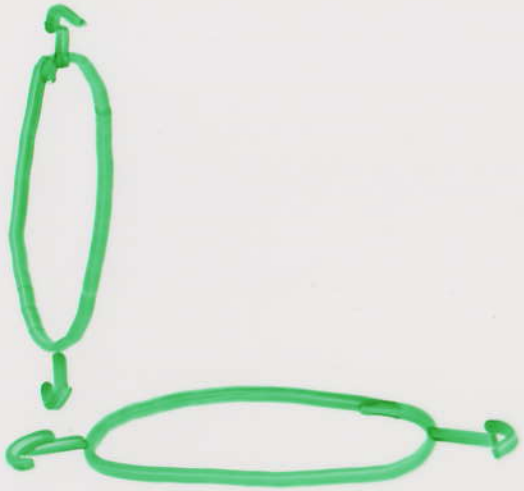
ii) when interact at large momentum.

Suggests look at high density or high temperature.

NB: condensed matter physics teaches us that phases may be far from simple even for α as small as $\frac{1}{137}$.

EXPLORING *the* PHASES *of* QCD





The interesting
"in-between regions"

$T \neq 0$; $\mu = 0$

- vertical axis
- we know a <u>lot</u> from lattice QCD. eg →
- QCD describes a transition

FROM	TO
gas of hadrons	plasma of quarks and gluons
with chiral symmetry badly broken	with chiral sym. almost restored.

- $T_c \simeq 175 \pm 15$ MeV
- The transition is a <u>smooth crossover</u>, like ionization of a gas, occurring in a narrow range of T.

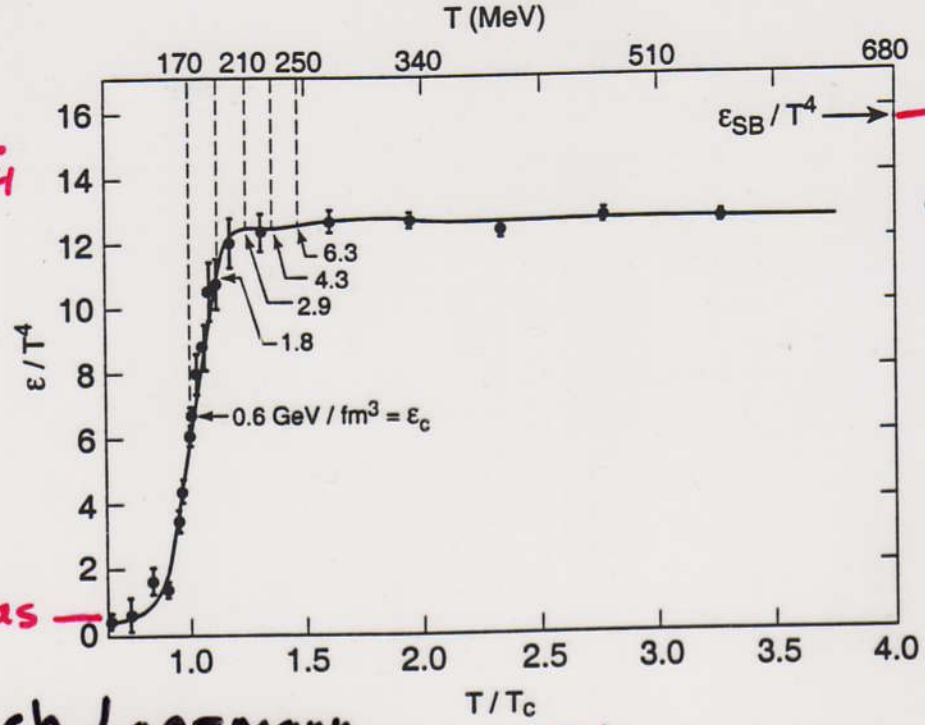
<u>IF</u> $m_s \gtrsim \frac{1}{5} m_s^{physical}$, and so in nature

NB: In world with $m_u = m_d = m_s$, crossover if $m_q \gtrsim \frac{1}{15} m_s^{physical}$

Bielefeld
Columbia

T(MeV), assuming $T_c = 170$ MeV.
(estimate is $140 < T_c < 190$)

$\frac{\varepsilon}{T^4}$



ideal QGP

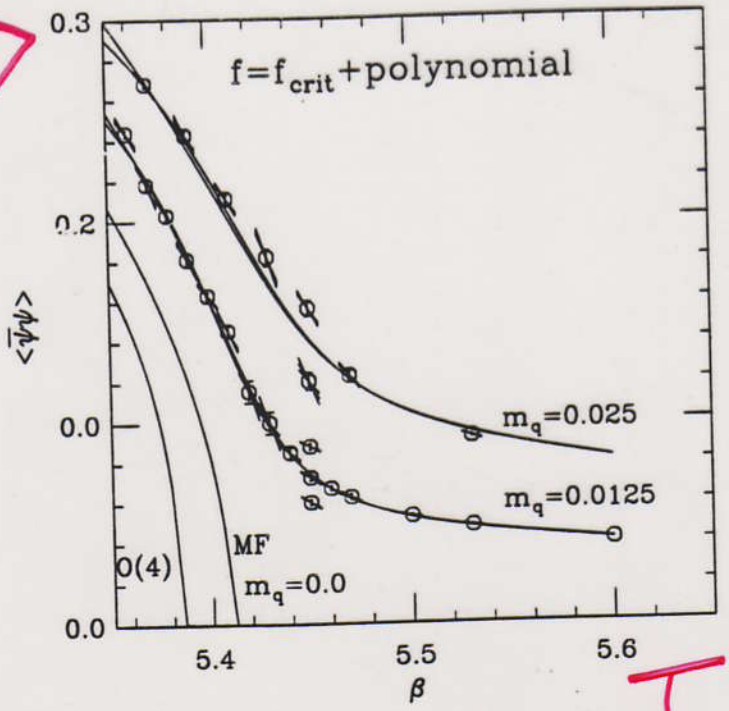
pion gas

DECONFINEMENT
(IONIZING THE HADRONS)

Karsch Laermann Peikert (Heine)

T/T_c

$\langle\bar{\psi}\psi\rangle$



Blum DeTar MILC collab.

CHIRAL SYMMETRY RESTORATION
(MELTING THE VACUUM)

ON THE LATTICE

$N_f = 2$
$m_q \neq 0$
$\therefore$ smooth crossover

T
(funny units)

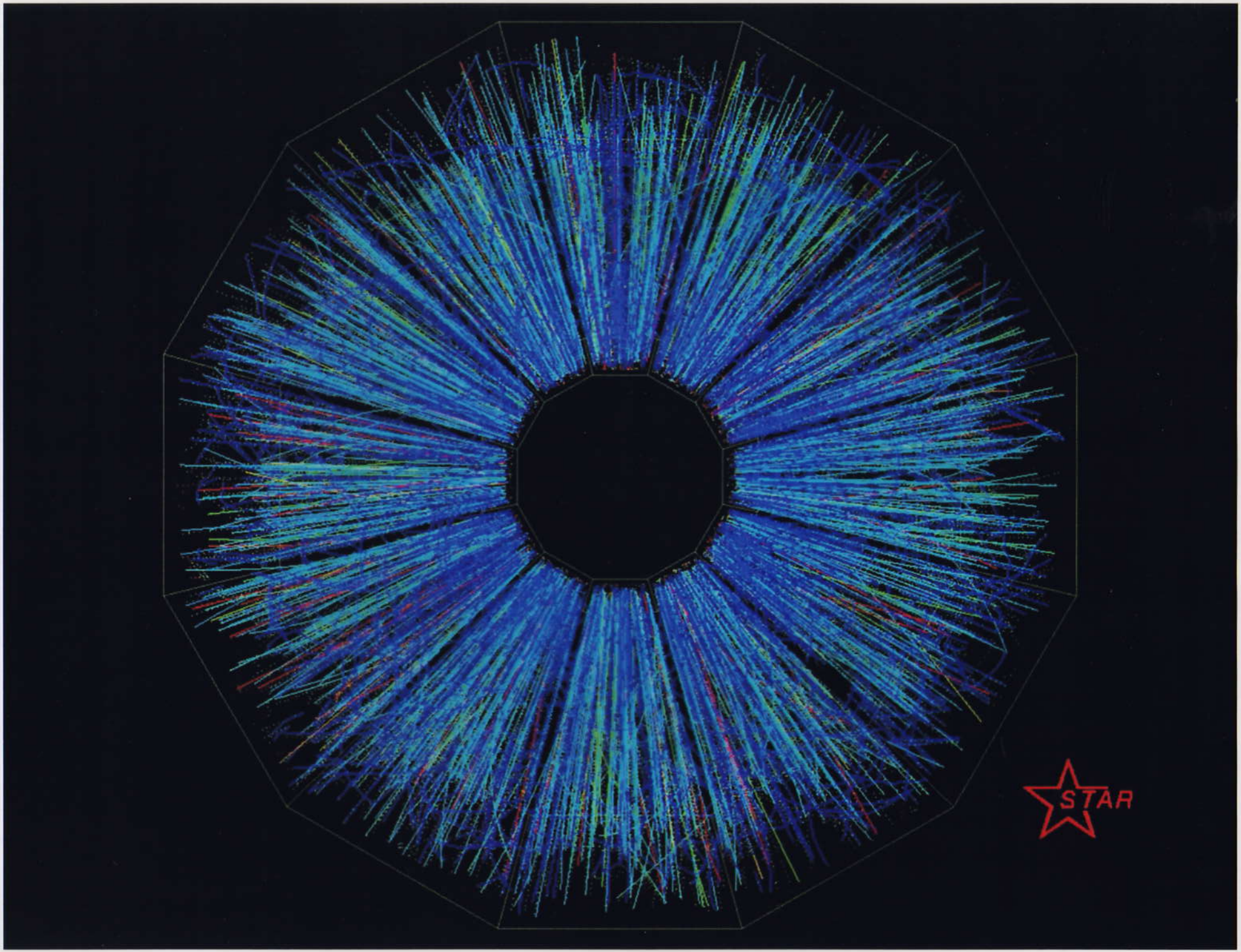
COSMOLOGICAL CONSEQUENCES?

Nobody has proposed an observable signature of the QCD transition in the early universe, if it is a crossover.

<u>BUT</u>: A sufficiently strongly first order transition messes up big bang nucleosynthesis, in a way inconsistent with data

<u>SO</u>: Cosmological "nonobservation" of a 1st order QCD transition is consistent with lattice QCD.

<u>AND</u>: If you want to probe the properties of hot quark matter, last seen microseconds after the big bang, you need experiments that recreate it.


STAR

HEAVY ION COLLISIONS: A BRIEF INTRODUCTION

- A picture worth 1000 words →
- Sequence of events:
  - i) collision leaves <u>lots</u> of gluons + quarks at mid-rapidity
  - ii) interaction → thermalization ??
    - must be tested experimentally
  - iii) if yes, hot fireball expands, cools, follows some track on phase diagram
  - iv) "Freezeout" (after which hadrons fly outwards in detector.) Much evidence from SPS + RHIC suggests final state at freezeout is expanding, ~equilibrated, hadron gas.
- What does higher $\sqrt{s}$ buy?
  - higher initial T, we hope
  - lower baryon #/entropy → lower μ
  - little change in freezeout T.

Chemical freeze–out in the T–μ_B plane

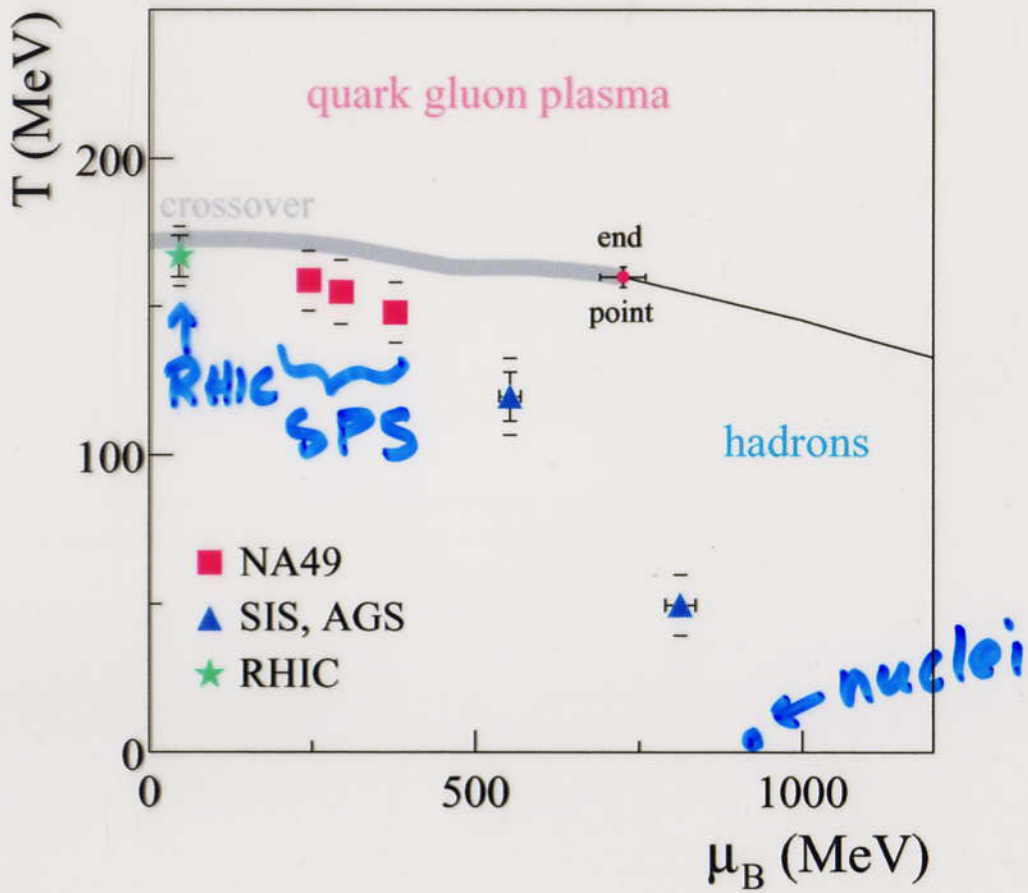
40 and 80 *A*GeV yields also fitted

	40 *A*GeV	80 *A*GeV	158 *A*GeV
T (MeV)	148 ± 2	155 ± 4	159 ± 2
μ_B (MeV)	377 ± 7	294 ± 15	244.5 ± 4.7
γ_S	0.75 ± 0.02	0.72 ± 0.03	0.82 ± 0.02
χ^2/NDF	14.8/4	10.4/4	23.5 / 11

$\sqrt{s}$: 9 12 17

fits by F. Becattini

- Freeze–out parameters on a (relatively) smooth curve
- Curve approaches phase boundary in the SPS energy range
- Even at RHIC, the parameters do not enter QGP–phase



Cross–over line from Z. Fodor, S.D. Katz hep–lat/0204029

EXPLORING QGP PROPERTIES

"Making QGP" is not a yes/no question:
No sharp boundary between hadrons, QGP.

Goal of RHIC: create matter ① that is above the crossover ② and study its properties. ③

①: RHIC data (on v_2) tell us interactions sufficient to yield ~equilibrated matter, expanding collectively as a fluid, by a time ~ 0.6 – 1 fm. After that hydrodynamics (ideal hydro; zero mean free path; ideal liquid not ideal gas) describes "bulk" of particles ($p_T \lesssim 1$-2 GeV) well.

②: RHIC data (dE_T/dy) tell us

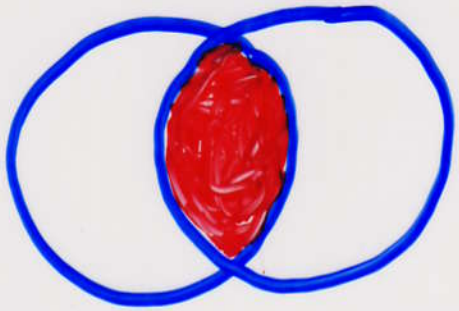
$$\varepsilon(1\,\text{fm}) \underset{\text{NB}}{>} 5\,\text{GeV/fm}^3 \Rightarrow \text{above crossover}$$

So, on to ③.....

TOWARD MEASURING SHEAR VISCOSITY

Elliptic flow indicates extent of early equilibration.

Look at non-head-on collisions:

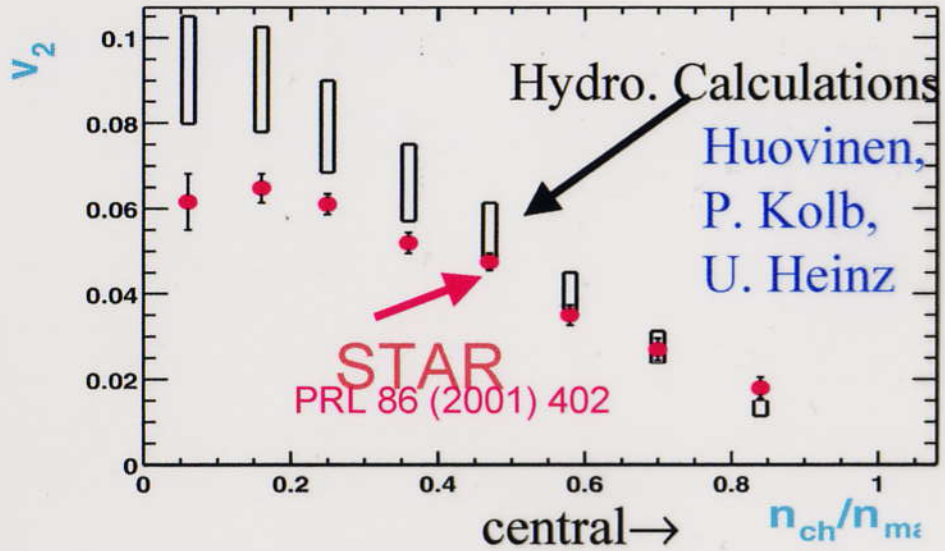


If just lots of p-p collisions followed by free streaming, then final state momenta uniformly distributed in azimuth angle ϕ.

If interaction → equilibration → pressure, pressure gradients → collective flow.

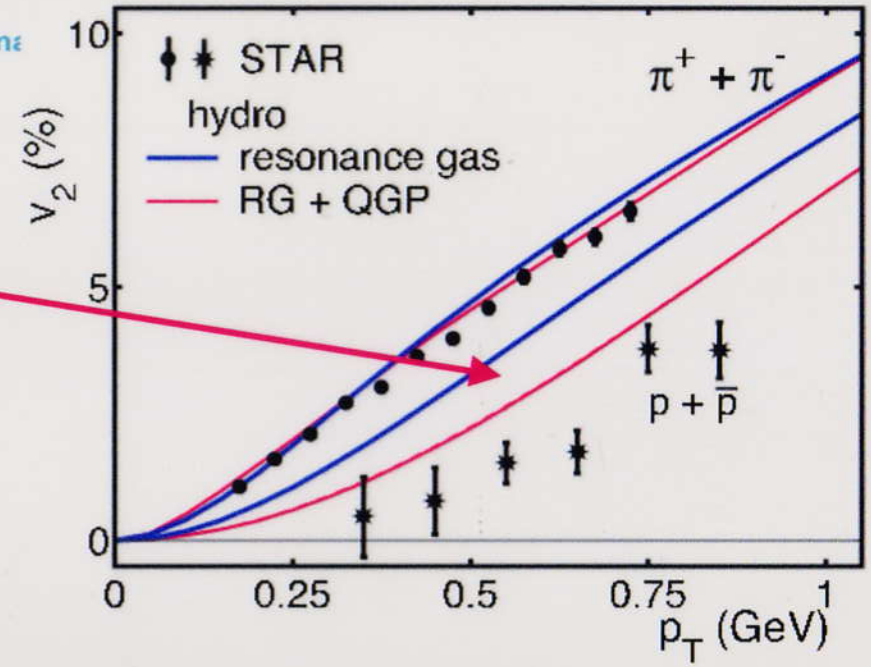
If this happens early, before circularizes by free streaming, then nonzero $V_2 \sim \langle \cos 2\phi \rangle$.

v_2 predicted by hydrodynamics



pressure buildup → explosion happens fast → early equilibration !

Hydro can reproduce magnitude of elliptic flow for π, p. BUT
must add QGP to hadronic EOS!!



Similar conclusion reached by CM Ko, et al., Kapusta, et al., Bleicher, et al., among others…

talk by B. Jacak

- Ideal hydrodynamics based on <u>assumption</u> of local eqbm.
- Hydro never agreed with data before RHIC. (At SPS, $v_2^{data} \sim \frac{v_2^{hydro}}{2}$
- At RHIC, hydro does good job of describing v_2, spectra for $P_T < 1\text{-}2\,GeV$
- <u>MEANS</u>: "hydro works" by $t \sim 0.6\text{-}1$ fm

  Heinz Kolb
- Challenge to theory: how can ~equilibration occur so quickly? Strong interactions? Strong color fields → plasma instabilities?

  Mrowczynski Rebhan Romatschke Strickland Arnold Moore Yaffe, ...
- <u>MEANS</u>: "small" shear viscosity η.

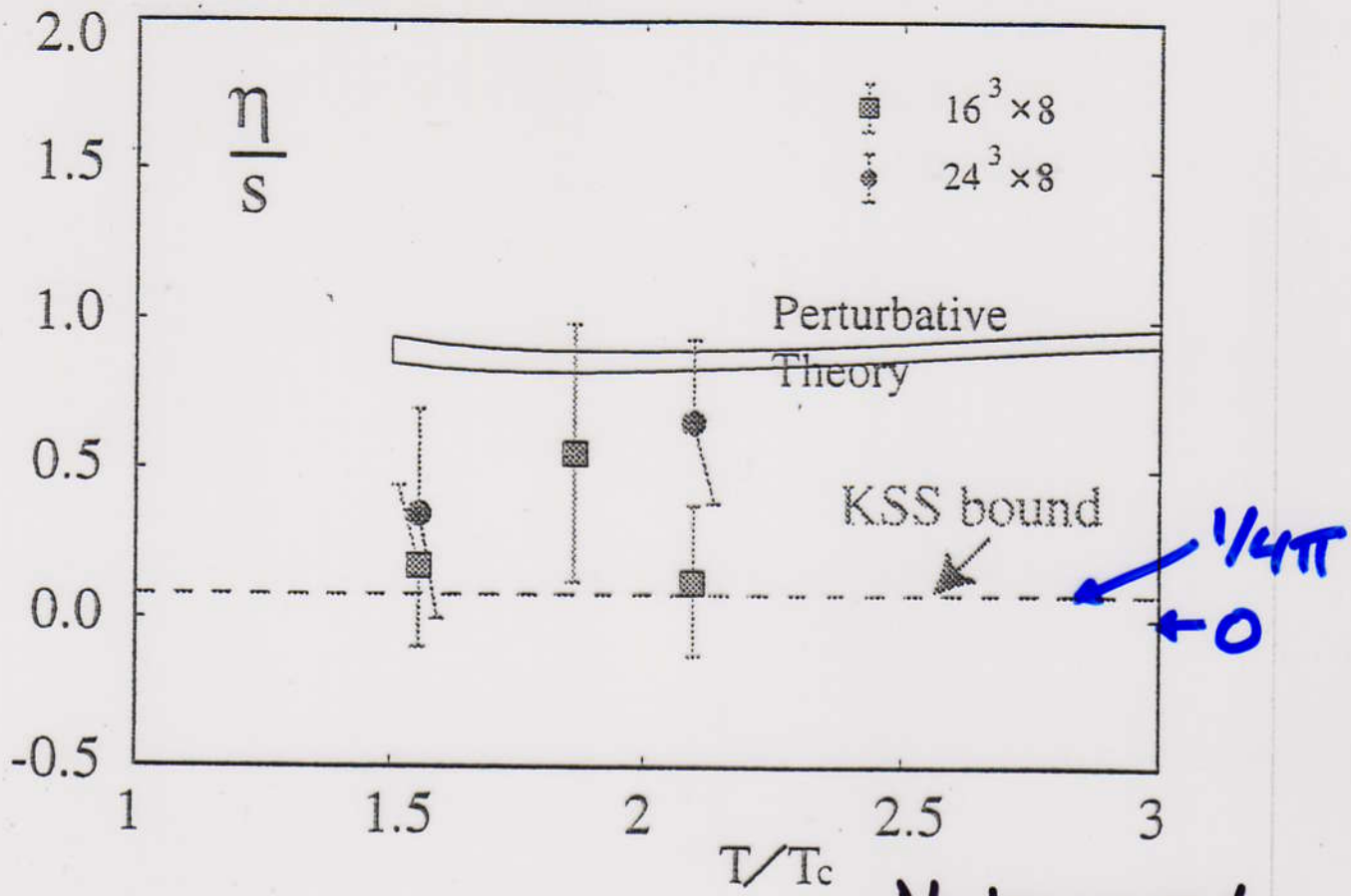
  Teaney: $\eta/s < \mathcal{O}(\frac{1}{10})$

  [cf water: $\eta/s > 10$]

CHALLENGES: Real extraction of η requires hydro calculations with $\eta \neq 0$.

Muronga; Heinz Song Chaudhuri

η/s FROM LATTICE QCD?



Nakamura & Sakai

Not an ab initio lattice calculation; doing that for transport coefficients is hard. (See, e.g., Petrecky + Teaney)

Parametrize Minkowski space spectral function with few-parameter ansatz, fit those parameters to lattice calculation of Euclidean correlation function.

See Also Gavai + Gupta. They extract electrical conductivity this way, then use kinetic theory to → $\eta/s \sim 0.2$

RHIC experiments seem to be studying the properties of a

QUARK - GLUON LIQUID

In a gas, or a weakly-coupled plasma,

mean free paths $\gg$ spacing between particles ($\sim \frac{1}{T}$ for us)

$\rightarrow$ quasiparticles with width $\ll$ mass.

$\rightarrow$ $\eta/s \gg 1$

In a liquid, m.f.p. $\lesssim$ spacing, no well-defined quasiparticles.

NB: for T large enough, pQCD works, mfp $\sim \frac{1}{\alpha^2 T}$, $\frac{\eta}{s} \sim \frac{1}{\alpha^2}$, QGP not a liquid anymore.

Should we be surprised if/that the QGP turns out to be liquid-like?

1) No. At $T \sim$ few T_c, coupling not small

2) But .... Lattice shows ε/T^4 reaches 80% of its value in an ideal-gas-QGP (ie noninteracting) already just above T_c. Doesn't this imply interactions are "just a 20% correction"???

3) $N=4$ SUSY QCD can teach us a lesson:

- $\varepsilon/T^4 = 75\%$ of its value in a noninteracting SUSY-QGP

Gubser Klebanov Tseytlin

- interactions <u>very</u> strong.

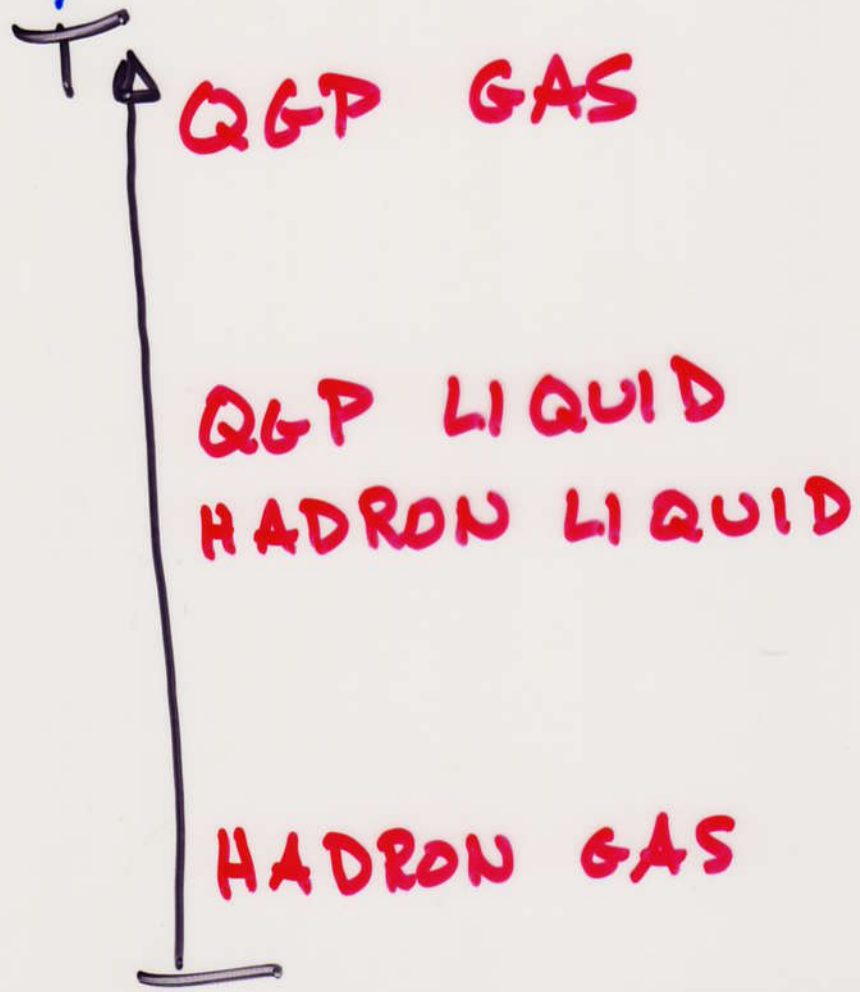
Policastro Son Starinets $\eta/s = \frac{1}{4\pi} \rightarrow$ m.f.p. ~ spacing

Kovtun Son Starinets - a liquid with lower viscosity per entropy than water

- ideal hydro!

- Teaney uses v_2 data to suggest η/s of real world QGP ~ as small.

So, a posteriori, (ie after the data) it is not surprising to find a QGP liquid. In fact, given that the transition is a crossover, it probably has to be this way:

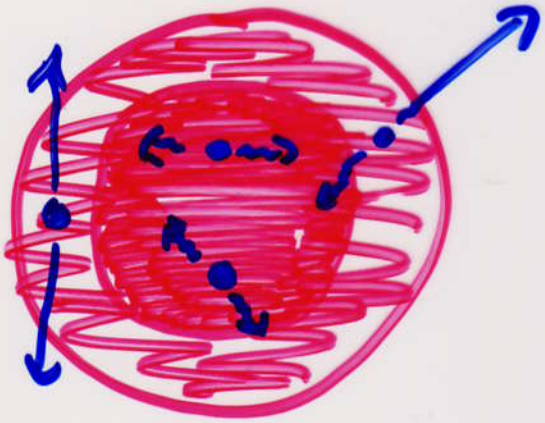


Also a posteriori (in this case, after the string theorists) we realize that $\frac{\varepsilon}{T^4} = 80\%$ of noninteracting is closer to 75% (strong coupling) than to 1.

TOWARD MEASURING OPACITY, AND PERHAPS v_{sound}, AND BOUNDING ε

"Jet quenching": RHIC data suggests that the rare high-p_T particles produced in initial hard scatterings are efficiently stopped → matter is "opaque".

Picture suggested is:



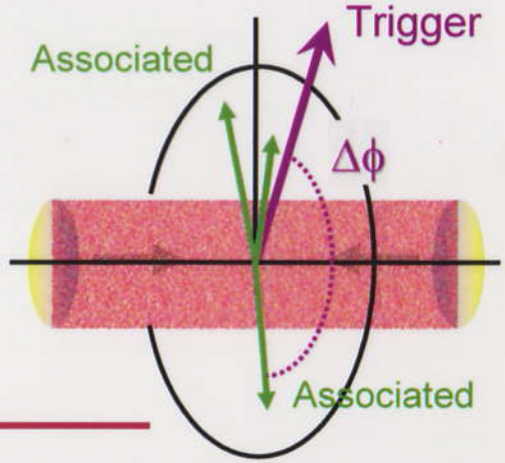
Ingoing, and interior, jets quenched.

Should see <u>some</u> back to back jets at any p_T, and more and more at higher p_T.

Evolution of $\Delta\phi$ correlations at RHIC

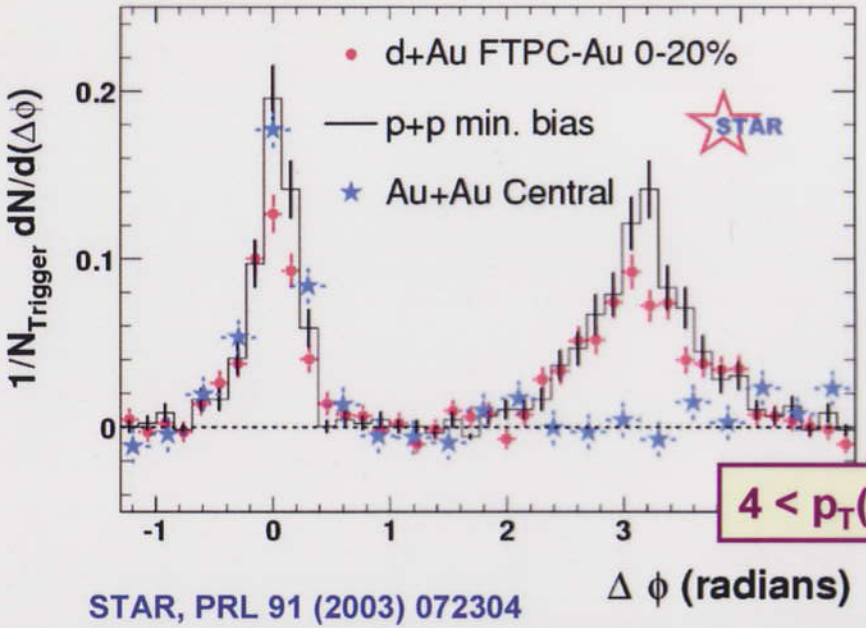
$\Delta\phi$ correlations

- "Trigger-associated" technique valuable for tagging jets in high-multiplicity environment (vs. jet-cone algorithms)
- Probes the jet's interaction with the QCD medium
- Provides stringent test of energy-loss models



Higher p_T → Away-side suppression

p_T(assoc) > 2 GeV/c

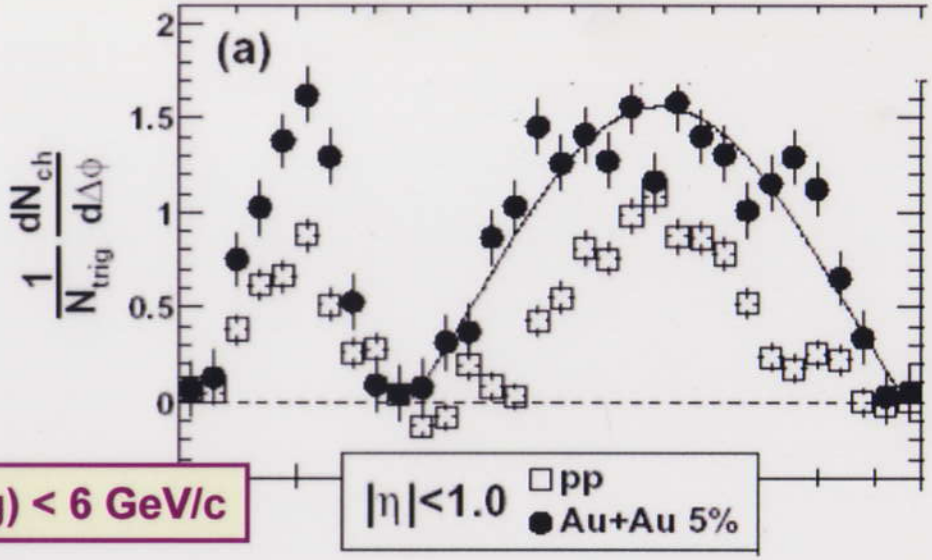


$4 < p_T(\text{trig}) < 6$ GeV/c

STAR, PRL 91 (2003) 072304

Lower p_T → Away-side enhancement

p_T(assoc) > 0.15 GeV/c



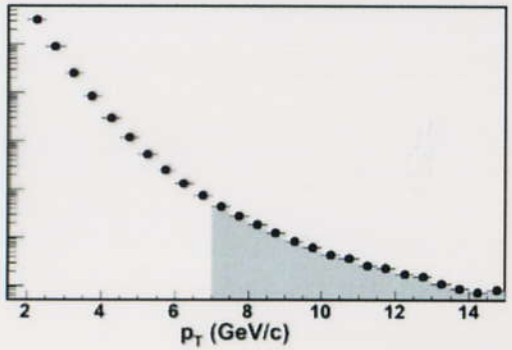
STAR, nucl-ex/0501016

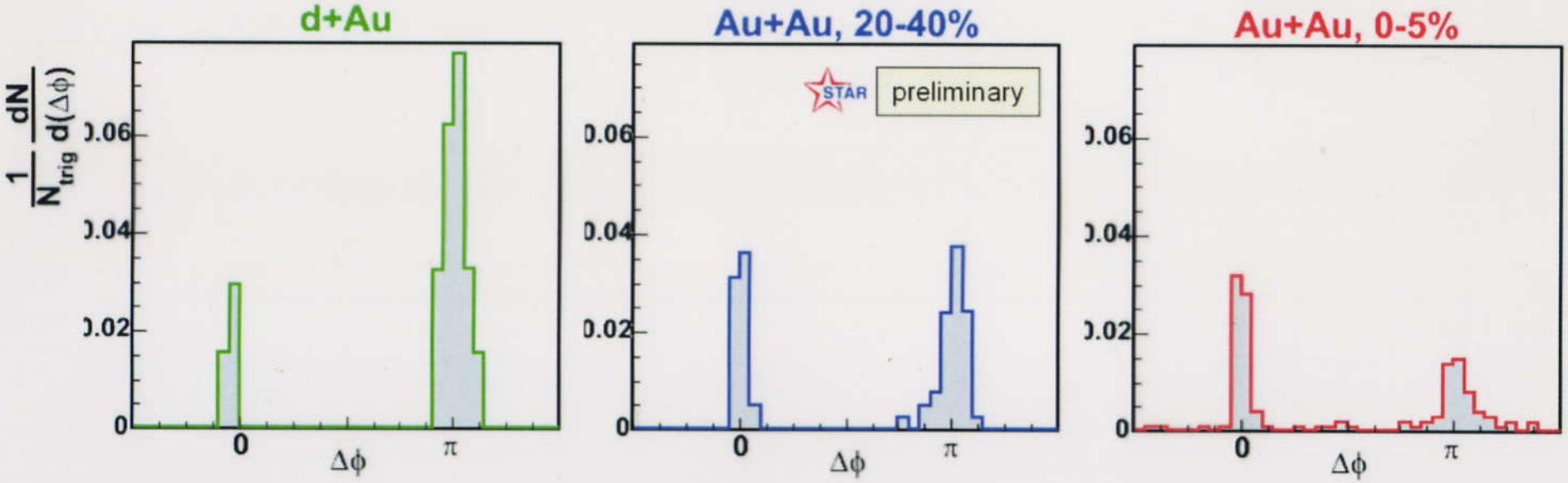
Emergence of dijets w/ increasing p_T(assoc)

- $\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15$ GeV/c

$p_T(\text{assoc}) > 7$ GeV/c





- Narrow peak emerges cleanly above vanishing background

v_{sound}?

Some reports of a "Mach cone" on the away side, where the supersonic jet was heading before it was quenched. If this persists as the data is further analyzed (via 3-particle correlations) then measure opening angle of the sonic boom $\rightarrow$ v_{sound}.

CAN WE MEASURE (OR BOUND) ν AND DEMONSTRATE DECONFINEMENT?

$$\nu \sim \frac{\varepsilon}{T^4} \sim \frac{s}{T^3} \sim \frac{s^4}{\varepsilon^3}$$

We have a lower bound on $\varepsilon(1\,\text{fm})$.

Can we get upper bound on $T(1\,\text{fm})$?

Challenge to exp. + th. (Υ's? J/ψ's?)

We can estimate $s(1\,\text{fm})$ from final state entropy, assuming equilibration before 1 fm. $s(1\,\text{fm}) = 33 \pm 3\ \text{fm}^{-3}$ Muller KR

(Error theory-dominated, improvable)

Now that jets seen to be punching through, can we use the whole suite of jet quenching observables to put an upper bound on $\varepsilon(1\,\text{fm})$?

Challenge to theory.

Motivation: if you could show $\varepsilon(1\,\text{fm}) < 7\ \frac{\text{GeV}}{\text{fm}^3}$ you would have shown $\nu > 26 \pm 8$.

$T \neq 0$; $\mu \neq 0$; μ/T NOT LARGE

$\mu \neq 0 \rightarrow$ complex Euclidean action.

$\rightarrow$ sign problem that makes difficulty of standard Monte Carlo $\sim e^V$.

Nevertheless, we are learning about this regime from lattice calculations that rely on smallness of μ/T.

These methods may be used to locate the...

CRITICAL POINT

A 2nd order point in the phase diagram where a line of 1st order transitions end. (Location is sensitive to quark masses. Moves leftward as masses $\downarrow$.)

LOCATING THE CRITICAL POINT

Range of estimates: $\ulcorner NB: \mu_B = 3\mu \lrcorner$

$$\frac{\mu_B^{\text{Endpoint}}}{T_c(\mu=0)} \sim 1, \sim 2, \sim 3$$

Gavai Gupta — Fodor Katz — Ejiri et al

Error estimates uncertain and clearly still large. Not at all like calculations of Tc. Yet.

Race between lattice QCD and experiment to locate the critical point....

HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

Increasing $\sqrt{s}$ → decreasing μ_B.

ENERGY:	AGS	SPS	RHIC
$\mu_B^{\text{Freezeout}}$:	550 MeV ←		→ 30 MeV

Vary $\sqrt{s}$, and hence μ_B, and look for enhancement (rise & then fall) of event-by-event fluctuations of:

i) mean p_T of low p_T pions Stephanov KR Shuryak

ii) observables that are proxies for baryon number, like # of protons − $\bar{p}$.

Hatta Ikeda, Hatta Stephanov

Seen on lattice by Bielefeld-Swansea. →

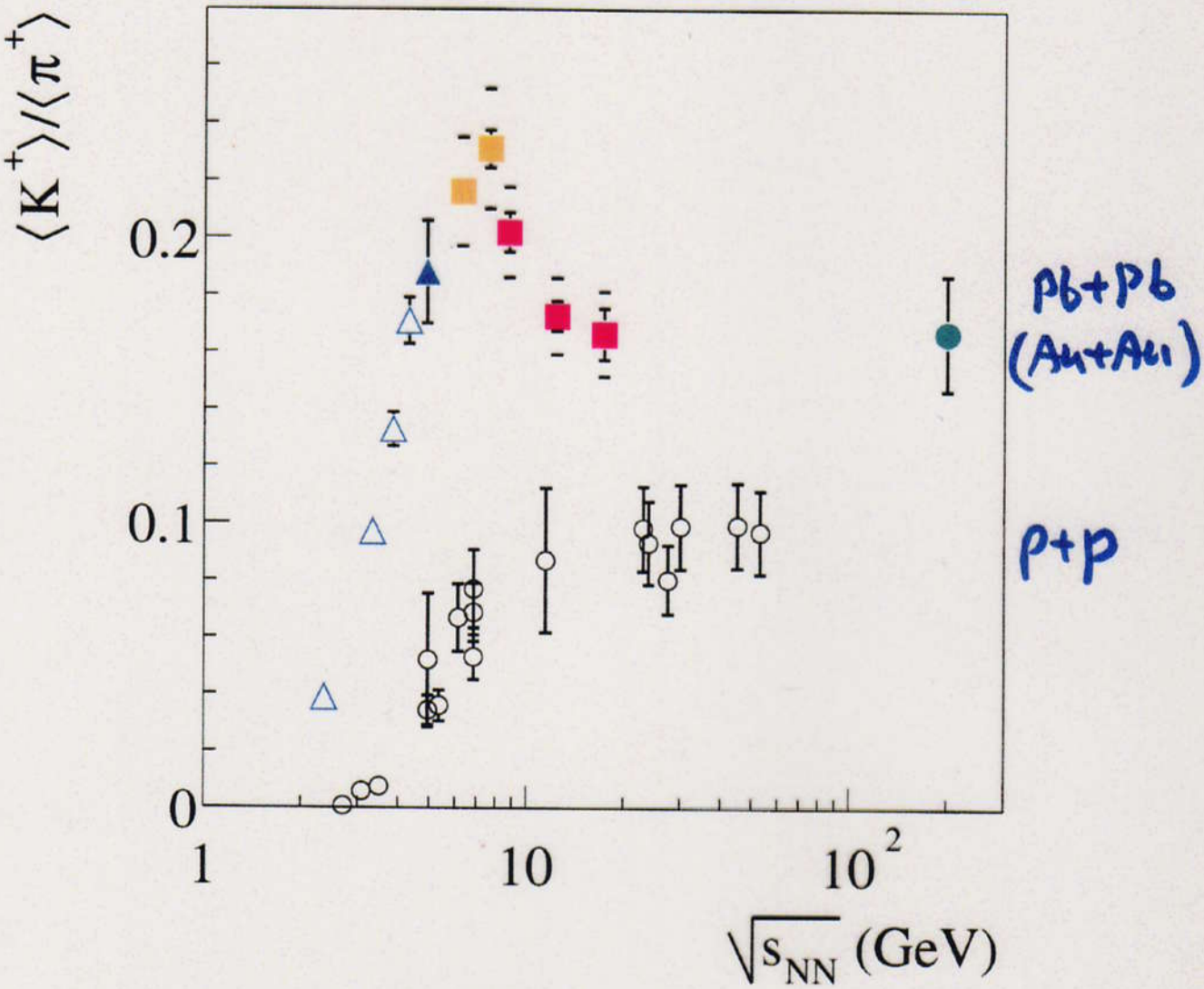
iii) Particle ratios ???

- will better survive late time hadron gas than p_T-fluctuation

And.....

Here is another quantity – not an e-by-e fluctuation – that varies nonmonotonically with $\sqrt{s}$...

Hadron Multiplicities ($\bar{s}$-QUARK CARRIER)

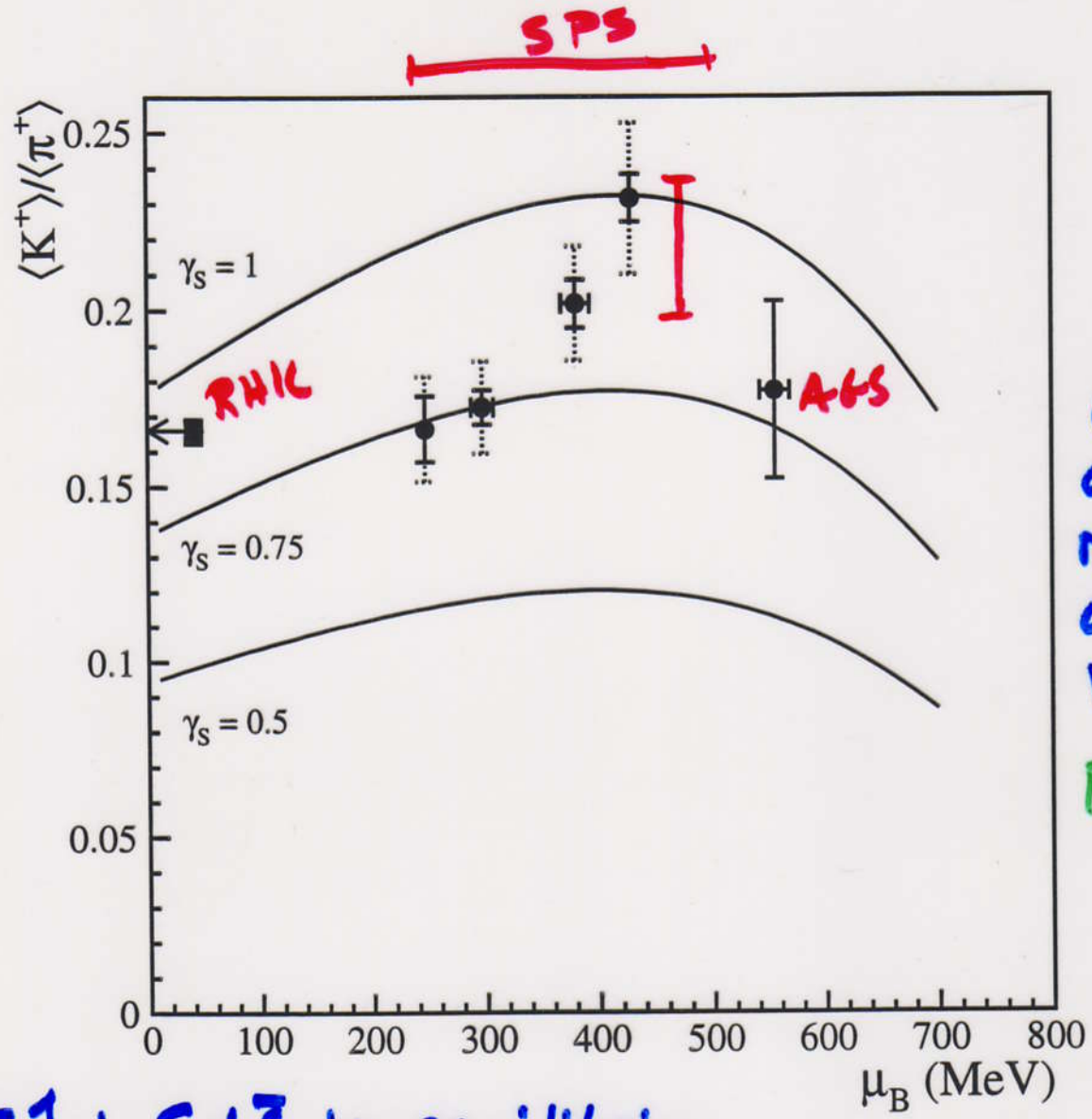


THE HORN

Talk by M. Gazdzicki, QM04, reporting NA49 results.

(Horn also seen in $\frac{K^- + \Lambda}{\pi^-}$.)

FIG. 13: Measured $\langle K^+\rangle/\langle\pi^+\rangle$ratio as a function of the fitted baryon-chemical potential. The full square dot is a preliminary full phase space measurement in Au-Au collisions at $\sqrt{s}_{NN} = 200$ GeV [37] and the error is only statistical; the arrow on the left signifies that its associated baryon chemical potential is lower than that estimated at $\sqrt{s}_{NN} = 130$ GeV [11] used here. For the SPS energy points the statistical errors are indicated with solid lines, while the contribution of the common systematic error is shown as a dotted line. Also shown the theoretical values for a hadron gas along the fitted chemical freeze-out curve shown in fig. 11, for different values of γ_S.



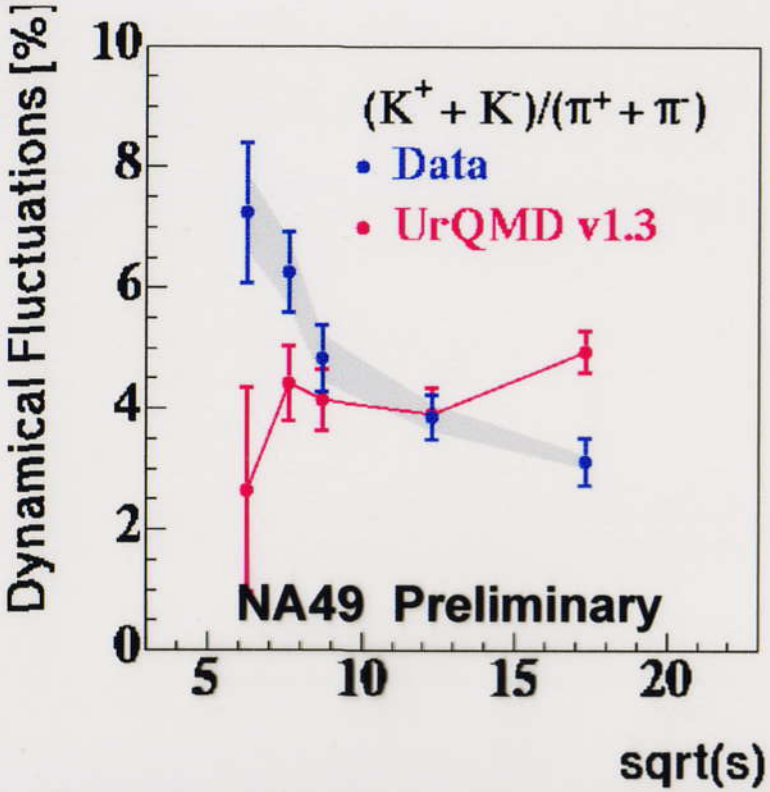
Curves are thermal ratios, calculated by Becattini et al.

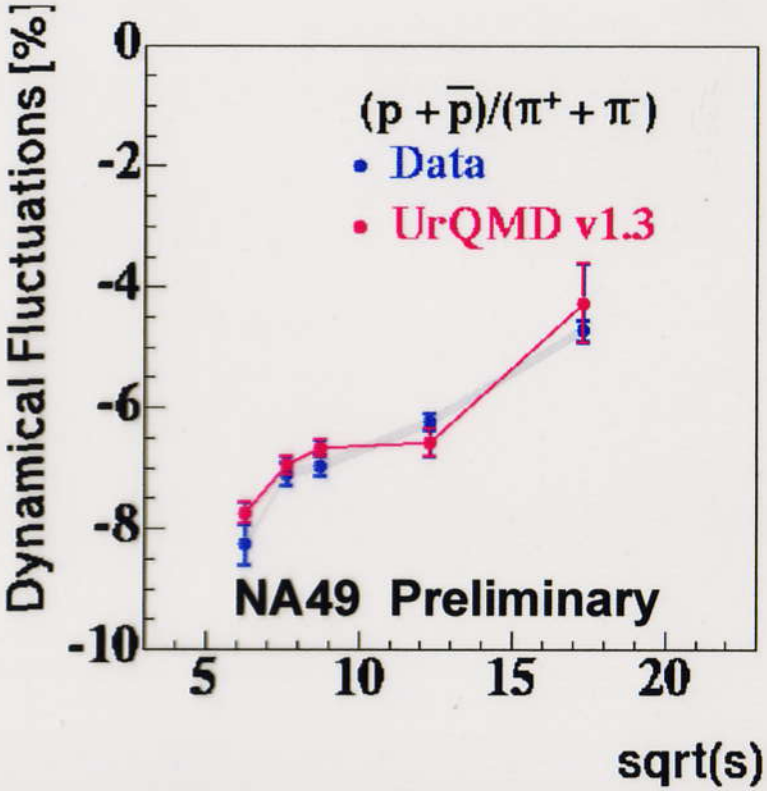
$\gamma_s = 1$: $s+\bar{s}$ in equilibrium

$\gamma_s = 0.75$: $s+\bar{s}$ is 75% of eqbm value

To explain "horn", need $\gamma_s = 1$ at horn and $\gamma_s \sim .7 - .8$ on either side of horn.

Summary





- K/π fluctuations increase towards lower beam energy
  - **Significant enhancement over hadronic cascade model**
- p/π fluctuations are negative
  - **indicates a strong contribution from resonance decays**

Intriguing...

Large e-by-e fluctuations at $\mu_B \sim 400-450$ MeV, in K/π.

Are the fluctuations dominated by low p_T π + K?

Why no p/π fluctuations?

Majumder + Koch suggest this points towards first order, not critical point.

Are there p_T-fluctuations?

cf: Fodor Katz $\rightarrow \mu_B^{\text{Endpoint}} \sim 360$ MeV $\rightarrow \sqrt{s} \sim 8$ GeV

Gavai Gupta $\rightarrow \mu_B^{\text{Endpoint}} \sim 180$ MeV $\rightarrow \sqrt{s} \sim 25$ GeV

Motivation for an energy scan at low energies at RHIC.

Goal: Mark a ● on phase diagram.

HIGH DENSITY & LOW TEMPERATURE

Whereas at high T entropy wins
→ quark-gluon plasma with symmetries of QCD Lagrangian manifest

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

- At large enough μ (to be defined below) we have <u>answers</u>.
- At large but notso large μ, we have a <u>puzzle</u>, and <u>hints</u>.
- How can we use astrophysical observations of compact stars to provide answers?

WHY COLOR SUPERCONDUCTIVITY?

Large μ → quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom → <u>weak</u> interactions between quarks at Fermi surface.

<u>BUT</u> <u>any</u> attractive interaction, no matter how weak, →

COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color $\bar{3}$.

(no need to resort to phonons; ∴ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,

⇒ - electric & color currents superconduct
- mass for photon & (some) gluons (?)
- Meissner effects. (Magnetic & color magnetic fields excluded.)

Barrois; Bailin & Love

GAP AND T_c

Much work (that I will not review) suggests that @ $\mu_q \sim 500$ MeV [$\sim 10 \times$ nuclear density]

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ THIS is high T_c s.c.!

Two classes of methods ~ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R, Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

ii) weak-coupling QCD calculations, valid for $\mu \to \infty$; $g \to 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^9$ MeV K.R, Shuster)

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{\text{Schafer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; K.R, Shuster; Rischke, Wong; ....}} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{\text{Son}}$$

[$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:

$$\rightarrow 1 = g^2 \underbrace{\ln \frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln \frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

CFL

In cold quark matter, quarks near their Fermi surfaces pair
→ color superconductivity

Pattern of pairing:

$$\langle \psi^{\alpha}_{a} \underbrace{C\gamma_5}_{\text{Lorentz scalar}} \psi^{\beta}_{b} \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1} \epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2} \epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3} \epsilon_{ab3}$$

← color

flavor

- antisymmetry in color + Dirac indices energetically favored; flavor antisym. forced by Pauli
- If density great enough that M_s can be neglected, $\Delta_1 = \Delta_2 \simeq \Delta_3$
- All 9 quarks pair, maximizing condensation energy; leaves largest symmetry unbroken
- Demonstrated rigorously at asymptotic density.
- Unbroken symmetries all are color + flavor

$$
\begin{array}{c|cccccccccc}
 & ru & gd & bs & rd & gu & bu & rs & gs & bd \\
\hline
ru & & -\Delta_3 & -\Delta_2 & & & & & & \\
gd & -\Delta_3 & & -\Delta_1 & & & & & & \\
bs & -\Delta_2 & -\Delta_1 & & & & & & & \\
rd & & & & & \Delta_3 & & & & \\
gu & & & & \Delta_3 & & & & & \\
bu & & & & & & & \Delta_2 & & \\
rs & & & & & & \Delta_2 & & & \\
gs & & & & & & & & & \Delta_1 \\
bd & & & & & & & & \Delta_1 &
\end{array}
$$

Define

$$
\tilde{Q} = \begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix} \text{for} \begin{matrix} u \\ d \\ s \end{matrix} + \begin{pmatrix} -2/3 \\ 1/3 \\ 1/3 \end{pmatrix} \text{for} \begin{matrix} r \\ g \\ b \end{matrix}
$$

and check $\tilde{Q} = 0$ for every pair in the condensate.

$\Rightarrow$ One linear combination of photon & gluon does <u>not</u> get "Meissnered".

$$U(1)_{EM} \times SU(3)_{color} \to U(1)_{\tilde{Q}}$$

COLOR-FLAVOR LOCKED QUARK MATTER

- occurs for $\mu \to \infty$, and at any μ if $m_s = m_u = m_d$
- all 9 quarks pair and are gapped
- superfluid
- chiral symmetry spontaneously broken, by a new mechanism (CFL).
  - $\Rightarrow$ "pions" and "kaons" lightest excitations
    - massless if $m_s = m_u = m_d = 0$
    - ~ few MeV mass ($\ll \Delta$) for real $m_{s,u,d}$.
    - "K_0" may condense
- Unbroken gauged $U(1) \to$ massless photon
- As long as $T <$ meson masses ~ few MeV:
  - Transparent insulator (neutral without electrons)
    - index of refraction and reflection/refraction coefficients known
  - Very small specific heat, neutrino emissivity, viscosity. Good thermal conductor
- All these properties, and more, rigorously calculable in $\mu \to \infty$, $g \to 0$ limit. Chiral symmetry breaking and all its consequences understood at high density.
- Occurs in nature wherever $\mu > m_s^2/2\Delta$. (density depend.)
- What are the properties of quark matter at lower density ???

WHAT CAN BE CALCULATED?

from QCD from first principles?

- At asymptotic densities, answer is "everything"; more than in any other circumstance in QCD.
  - in the CFL phase, there are no unresolved nonperturbative ambiguities: no gapless fermions; no massless gluons. No IR difficulties.
  - calculation of Δ is nonperturbative, but controlled by smallness of g.
  - analogues of confinement and chiral symmetry breaking are calculable at weak coupling.

- At potentially accessible densities, g not small. Means Δ cannot be calculated precisely (barring a major lattice QCD breakthrough.)

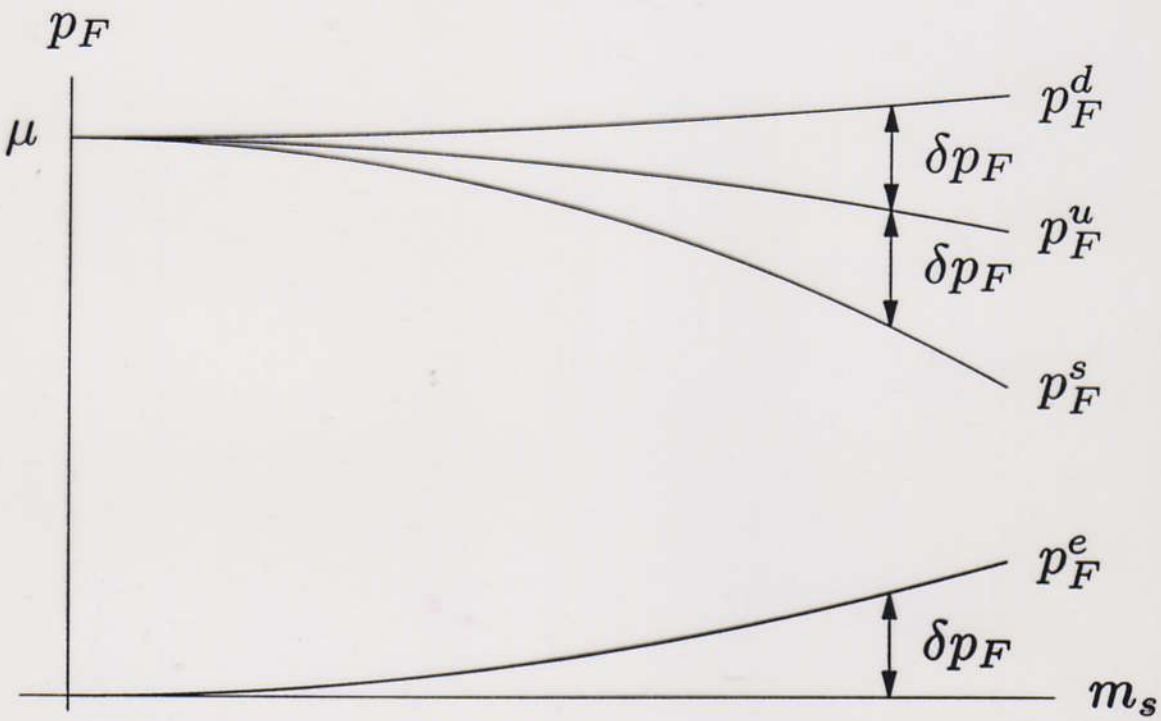
BUT: if you take Δ as given (ie treat as a parameter whose value known at order of magnitude level) then <u>many</u> physical properties calculable in terms of Δ.

Eg: specific heat, thermal conductivity, index of refraction, neutrino opacity, neutrino emissivity, shear viscosity, bulk viscosity, ...

[Many of these described within an effective field theory for the Goldstone bosons, whose parameters are determined by Δ.]

INTERMEDIATE DENSITY QUARK MATTER

- M_s important
- For orientation, consider noninteracting quarks, $m_u = m_d = 0$ $m_s \neq 0$, impose electrical neutrality and weak eqbm:



- In noninteracting quark matter, $\delta p_F \simeq \frac{m_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\frac{m_s^2}{4\mu} > \Delta$??
- Also, when CFL "breaks", no residual $\langle ud \rangle$ pairing either. Alford, KR

WHAT REPLACES CFL, AT LOWER μ?

We don't yet know.....

We do know:

- CFL pairing is unstable once $\mu < \frac{M_s^2}{2\Delta}$

  (Alford Kouvaris KR)

  and stable for larger μ.

- $\therefore$ If Δ large enough & M_s not too large, CFL quark matter is stable all the way down to transition to nuclear matter.

eg: $M_s = 300$ MeV
$\Delta > 125$ MeV
or
$M_s = 200$ MeV
$\Delta > 55$ MeV

QUESTIONS:

What if less symmetrically paired quark matter intervenes? Ie, what are properties of quark matter with $\mu < M_s^2/2\Delta$?

What are astrophysical consequences if neutron stars have CFL cores?

LESS SYMMETRICALLY PAIRED Q.M.

Gapless CFL Phase?

- 2nd highest density phase within a spatially uniform ansatz
- nice distinctive astrophysical signature (Alford Jotwani Kouveris Kundu KR)
- unstable to currents → inhomogeneity (Huang Shovkovy; Casalbuoni et al; Giannakis Ren; Fukushima; ...)

Crystalline Color Superconductivity?

- may be the answer, but:
- until recently, analyzed only in 2-flavor setting, without imposing neutrality
- potential for astrophysical signatures, (Alford Bowers KR) but not yet analyzed sufficiently to say how distinctive

FCC Crystal

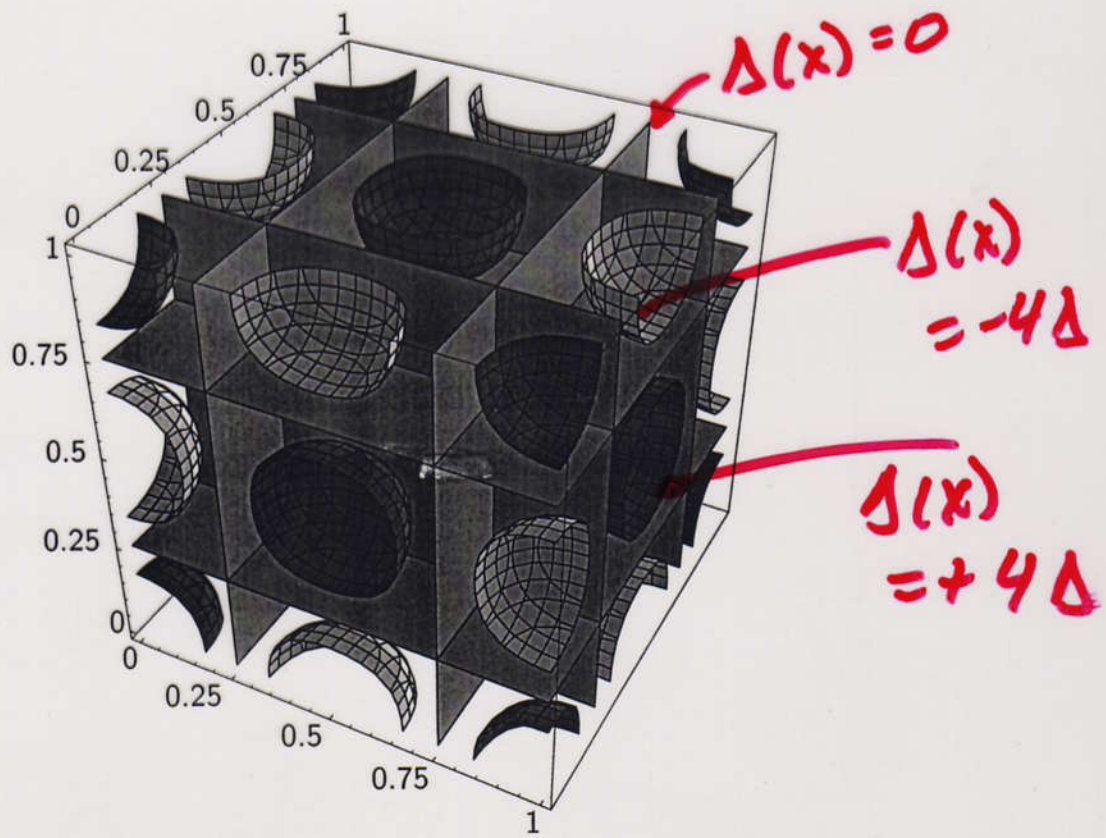
Favored according to Ginzburg-Landau analysis, that is not yet quantitatively reliable. Bowers KR

- The cube structure is the favored ground state: eight wave vectors pointing towards the corners of a cube, forming the eight shortest vectors in the reciprocal lattice of a face-centered-cubic crystal. The gap function is

$$\Delta(\mathbf{x}) = 2\Delta\left[\cos\frac{2\pi}{a}(x+y+z) + \cos\frac{2\pi}{a}(x-y+z) + \cos\frac{2\pi}{a}(x+y-z) + \cos\frac{2\pi}{a}(-x+y+z)\right]$$

$\Delta \sim \Delta_{CFL}$

A unit cell:



with contours $\Delta(\mathbf{x}) = +4\Delta$ (black), 0 (gray), -4Δ (white). Lattice constant is $a = \sqrt{3}\pi/|\mathbf{q}| \simeq 6.012/\Delta_0$.

Sum of 8 currents; zero net current.

OUTLOOK

As in 2-flavor case, GL approx. predicts:

- strong 1st order transition, with robust Δ, Ω.
- and therefore tells us GL not quantitatively reliable.

We have not yet understood enough cases to identify most favorable structures.

Current status: nobody has shown that a crystalline phase can have lower energy than gCFL throughout gCFL window, but the gCFL instability points in that direction, and preliminary GL analyses are encouraging.

OUTLOOK AND IMPLICATIONS

CRYSTALLINE SUPERFLUIDITY

- A two species version of crystalline superfluid may be created in gases of ultracold fermionic atoms
  Combescot; Son Stephanov
  - trap 2 hyperfine states of atom;
  - arrange strong attractive interaction between 2 "species". (Done via a Feshbach resonance.)
  - load the trap with different number densities for 2 "species".

VORTEX PINNING & PULSAR GLITCHES

- Rotate the crystal; what happens? Alford Bowers KR
  Vortices? Vortices pinned at intersections of crystal's nodal planes?
- If there are pinned vortices, the presence of a layer of crystalline color superconducting quark matter within neutron stars could make this layer a locus for Pulsar Glitches.

ASTROPHYSICAL CONSEQUENCES IF NEUTRON STARS HAVE CFL CORES

- For given M, R a little smaller. But, uncertainty in R still dominated by nuclear outer layer. Alford Reddy
- At a sharp interface, big density step. $\xrightarrow{?}$ LIGO signal Alford K.R. Reddy Wilczek
- If spherical stars have CFL cores but oblate stars do not, → unusual spin-up history. Glendenning, Weber; Blaschke, Grigorian, Pagazyan
- Transparent insulator. → $\vec{B}$ in core not in flux tubes; not frozen. → $\vec{B}$ evolution governed by outer layer.
- For $T <$ few MeV:
  - very small specific heat, neutrino emissivity, neutrino opacity. Page Prakash Lattimer Steiner Jaikumar Prakash Schaefer
  - superfluidity → very large thermal conductivity Shovkovy Ellis

  ⇒ cooling of star controlled by nuclear outer layer
- During supernova, $T \sim$ tens of MeV > meson mass
  → mesons emit and scatter neutrinos Reddy Sedrakian Tachibana; Carter Reddy
  - and, also, may be phase transitions

  $\xrightarrow{?}$ signals in time distribution of supernova ν
- Bare quark star would be nice. **NOT** seen...

GOALS

PUZZLE: If non-CFL quark matter intervenes between CFL & nuclear, what are its properties?

HINTS: gCFL instability $\overset{?}{\Rightarrow}$ crystalline condensate

COMING: neutral, 3-flavor crystalline color superconductor, with realistic crystal structure: does it have lower energy than gCFL?

LONGER TERM: Improve calculations of properties and consequences of these phases, allowing observations to rule their presence within neutron stars out or in. Eg:

- pinning force & shear modulus of X-tal $\overset{?}{\Rightarrow}$ glitches
- almost no limit to possible improvement in calculation of CFL properties
- new data coming on M, R, ν-cooling, SN-ν, LIGO, ...

NEWS

Nice, Splaver, Stairs, Löhmer, Jessner, Kramer, Cordes
astro-ph/0508050 (~~appeared this week~~)

A pulsar (named PSR J0751+1807) with mass:

$M = 2.1 \pm 0.2$ solar masses

$1.6 < M < 2.5$ at 95% confidence

- A 3.5 ms pulsar in a 6.3 hr orbit around a 0.19 solar mass white dwarf.
- Over 10 years of observation, the 6.3 hr orbit has slowed by $19.6 \pm 2.5\ \mu s$! due to gravitational wave emission.
- Shapiro delay measured. No accretion, mass transfer, or X-ray emission.
- ie this is gold-plated, as clean as the best previous mass measurements. And, the error bar will shrink like $(1/\text{duration of observation})^{2.5}$
- cf: $M_{NS} \lesssim 2.3\ M_\odot$ (stiff nuclear E.O.S.)
- cf: M_{NS} with quark core $< (1.9-2.0)\ M_\odot$ Alford Baty Paris Redd,